B Notes

B.1 Notes to section 3 on quark masses

Collab.	Ref.	N_f	a [fm]	Description
HPQCD 14A	[1]	2+1+1	0.15, 0.12, 0.09, 0.06	Scale set through the Wilson flow parameter w_0.
FNAL/MILC 14A	[2]	2+1+1	$0.06, 0.09, 0.12, 0.15$	HISQ action for both valence and sea quarks. Absolute scale though f_π.
ETM 14	[3]	2+1+1	0.062, 0.082, 0.089	Scale set through f_π. Automatic $\mathcal{O}(a)$ improvement, flavour symmetry breaking: $(M^0_{PS})^2 - (M^\pm_{PS})^2 \sim \mathcal{O}(a^2)$. Discretization and volume effects due to the $\pi^0 - \pi^\pm$ mass splitting are taken into account through χPT for twisted mass fermions.

Table 60: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
QCDSF/UKQCD 15	[4]	2+1	0.07	Scale set through the gradient flow parameter w_0.
RBC/UKQCD 14B	[5]	2+1	0.063, 0.084, 0.114, 0.144	Scale set through M_Ω.
RBC/UKQCD 12	[6]	2+1	0.085, 0.113, 0.144	Scale set through M_Ω.
PACS-CS 12	[7]	1+1+1	0.09	Reweighting of PACS-CS 08 $N_f = 2+1$ QCD configurations with e.m. and $m_u \neq m_d$.
Laiho 11	[8]	2+1	0.06, 0.09, 0.15	MILC staggered ensembles [9], scale set using r_1 determined by HPQCD with Υ splittings, pseudoscalar decay constants, through r_1 [10].

Table 61: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
PACS-CS 10	[11]	2+1	0.09	cf. PACS-CS 08
MILC 10A	[9]	2+1		cf. MILC 09, 09A
BMW 10A, 10B	[12, 13]	2+1	0.054, 0.065, 0.077, 0.093, 0.116	Scale set via M_π, M_K, M_Ω.
RBC/UKQCD 10A	[14]	2+1	0.114, 0.087	Scale set through M_Ω.
Blum 10	[15]	2+1	0.11	Relies on RBC/UKQCD 08 scale setting.
PACS-CS 09	[16]	2+1	0.09	Scale setting via M_Ω.
HPQCD 09A, 10	[17, 18]	2+1	0.045, 0.06, 0.09, 0.12, 0.15	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT. See MILC 09 for details.
MILC 09A, 09	[19, 20]	2+1	0.045, 0.06, 0.09	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT.
PACS-CS 08	[21]	2+1	0.09	Scale set through M_Ω. Non-perturbatively $\mathcal{O}(a)$-improved.
RBC/UKQCD 08	[22]	2+1	0.11	Scale set through M_Ω. Automatic $\mathcal{O}(a)$-improvement due to appoximate chiral symmetry. $(\Lambda_{\rm QCD}a)^2 \approx 4\%$ systematic error due to lattice artifacts added.
CP-PACS/JLQCD 07	[23]	2+1	0.07, 0.10, 0.12	Scale set through M_K or M_ϕ. Non-perturbatively $\mathcal{O}(a)$-improved.
HPQCD 05	[24]	2+1	0.09, 0.12	Scale set through the $\Upsilon - \Upsilon'$ mass difference.
HPQCD/MILC/UKQCD 04, MILC 04	[25, 26]	2+1	0.09, 0.12	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT.

Table 61: (cntd.) Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
ETM 14D	[27]	2	0.094	Scale set through F_π, r_0, t_0 and w_0. Twisted Wilson fermions plus clover term. Automatic $\mathcal{O}(a)$ improvement.
RM123 13	[28]	2	0.098, 0.085, 0.067, 0.054	cf. ETM 10B
ALPHA 12	[29]	2	0.076, 0.066, 0.049	Scale set through F_K.
RM123 11	[30]	2	0.098, 0.085, 0.067, 0.054	cf. ETM 10B
Dürr 11	[31]	2	0.076, 0.072, 0.060	Scale for light-quark masses set through m_c.
ETM 10B	[32]	2	0.098, 0.085, 0.067, 0.054	Scale set through F_π.
JLQCD/TWQCD 08A	[33]	2	0.12	Scale set through r_0.
RBC 07	[34]	2	0.12	Scale set through M_ρ.
ETM 07	[35]	2	0.09	Scale set through F_π.
QCDSF/UKQCD 06	[36]	2	0.065-0.09	Scale set through r_0.
SPQcdR 05	[37]	2	0.06, 0.08	Scale set through M_{K^*}.
ALPHA 05	[38]	2	0.07-0.12	Scale set through r_0.
QCDSF/UKQCD 04	[39]	2	0.07-0.12	Scale set through r_0.
JLQCD 02	[40]	2	0.09	Scale set through M_ρ.
CP-PACS 01	[41]	2	0.11, 0.16, 0.22	Scale set through M_ρ.

Table 62: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
HPQCD 14A	[1]	2+1+1	$128_{\pi,5}$ (173_{RMS})	Sea quark masses linearly extrapolated/interpolated to physical values. m_s determined from physical m_s/m_c and m_c.
FNAL/MILC 14A	[2]	2+1+1	$128_{\pi,5}$ (143_{RMS})	Linear interpolation to physical point. The lightest RMS mass is from the $a = 0.06$ fm ensemble and the lightest Nambu-Goldstone mass is from the $a = 0.09$ fm ensemble.
ETM 14	[3]	2+1+1	180_{π^0} ($220_{\pi^\pm}$)	Chiral extrapolation performed through $SU(2)$ χPT or polynomial fit.

Table 63: Chiral extrapolation/minimum pion mass in determinations of m_{ud}, m_s and, in some cases, m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
QCDSF/UKQCD 15	[4]	2+1	205 (val.)	Expansion around the symmetric point $m_u = m_d = m_s$.
RBC/UKQCD 14B	[5]	2+1	139	NLO PQ $SU(2)$ χPT as well as analytic ansätze.
RBC/UKQCD 12	[6]	2+1	170	Combined fit to Iwasaki and Iwasaki+DSDR gauge action ensembles.
PACS-CS 12	[7]	1+1+1		cf. PACS-CS 08
Laiho 11	[8]	2+1	210 (val.) 280 (sea-RMS)	NLO $SU(3)$, mixed-action χPT [42], with N^2LO-N^4LO analytic terms.
PACS-CS 10	[11]	2+1		cf. PACS-CS 08
MILC 10A	[9]	2+1		NLO $SU(2)$ SχPT. cf. also MILC 09A, 09.
BMW 10A, 10B	[12, 13]	2+1	135	Interpolation to the physical point.
RBC/UKQCD 10A	[14]	2+1	290	NLO PQ $SU(2)$ χPT as well as analytic ansätze.
Blum 10	[15, 22]	2+1	242 (valence), 330 (sea)	Extrapolation done on the basis of PQχPT formulae with virtual photons.

Table 64: Chiral extrapolation/minimum pion mass in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
PACS-CS 09	[16]	2+1	135	Physical point reached by reweighting technique, no chiral extrapolation needed.
HPQCD 09A, 10	[17, 18]	2+1		cf. MILC 09
MILC 09A, 09	[19, 20]	2+1	177, 224	NLO $SU(3)$ RSχPT, continuum χPT at NNLO and NNNLO and NNNNLO analytic terms. The lightest Nambu-Goldstone mass is 177 MeV (09A) and 224 MeV (09) (at a =0.09fm) and the lightest RMS mass is 258MeV (at a =0.06fm).
PACS-CS 08	[21]	2+1	156	NLO $SU(2)$ χPT and $SU(3)$ (Wilson)χPT.
RBC/UKQCD 08	[22]	2+1	242 (valence), 330 (sea)	$SU(3)$ PQχPT and heavy kaon NLO $SU(2)$ PQχPT fits.
CP-PACS/JLQCD 07	[23]	2+1	620	NLO Wilson χPT fits to meson masses.
HPQCD 05	[24]	2+1	240	PQ RSχPT fits.
HPQCD/MILC/UKQCD 04, MILC 04	[25, 26]	2+1	240	PQ RSχPT fits.

Table 64: (cntd.) Chiral extrapolation/minimum pion mass in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ETM 14D	[27]	2	140	Charged/neutral pion mass breaking, $M^2_{\pi^{pm}} - M^2_{\pi^0} \sim \mathcal{O}(a^2)$, estimated to be $\simeq 20$ MeV.
RM123 13	[28]	2	270	Fits based on NLO χPT and Symanzik expansion up to $\mathcal{O}(a^2)$. $\mathcal{O}(\alpha)$ e.m. effects included.
ALPHA 12	[29]	2	270	NLO $SU(2)$ and $SU(3)$ χPT and $\mathcal{O}(a^2)$ on LO LEC.
RM123 11	[30]	2	270	Fits based on NLO χPT and Symanzik expansion up to $\mathcal{O}(a^2)$.
Dürr 11	[31]	2	285	m_c/m_s determined by quadratic or cubic extrapolation in M_π.
ETM 10B	[32]	2	270	Fits based on NLO χPT and Symanzik expansion up to $\mathcal{O}(a^2)$.
JLQCD/TWQCD 08A	[33]	2	290	NLO χPT fits.
RBC 07	[34]	2	440	NLO fit including $\mathcal{O}(\alpha)$ effects.
ETM 07	[35]	2	300	Polynomial and PQχPT fits.
QCDSF/UKQCD 06	[36]	2	520 (valence), 620 (sea)	NLO (PQ)χPT fits.
SPQcdR 05	[37]	2	600	Polynomial fit.
ALPHA 05	[38]	2	560	LO χPT fit.
QCDSF/UKQCD 04	[39]	2	520 (valence), 620 (sea)	NLO (PQ)χPT fits.
JLQCD 02	[40]	2	560	Polynomial and χPT fits.
CP-PACS 01	[41]	2	430	Polynomial fits.

Table 65: Chiral extrapolation/minimum pion mass in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
HPQCD 14A	[1]	2+1+1	2.5-5.8	3.7	
FNAL/MILC 14A	[2]	2+1+1	2.8-5.8	$3.9_{\rm RMS}(3.7_{\pi,5})$	Includes error estimate from NNLO SχPT.
ETM 14	[3]	2+1+1	2.0 - 3.0	2.7_{π^0} $(3.3_{\pi^\pm})$	FV effect for the pion is corrected through resummed NNLO χPT for twisted mass fermions, which takes into account the effects due to the $\pi^0-\pi^\pm$ mass splitting.

Table 66: Finite volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
QCDSF/UKQCD 15	[4]	2+1	$1.7, 2.2, 3.4$		Effective field theory used to extrapolate to infinite volume.
RBC/UKQCD 14B	[5]	2+1	$2.0, 2.7, 4.6, 5.4$	3.8	Uses FV chiral perturbation theory to estimate the error, which is deemed negligible and omitted.
RBC/UKQCD 12	[6]	2+1	2.7, 4.6	$\gtrsim 4.0$	Uses FV chiral perturbation theory to estimate the error.
PACS-CS 12	[7]	1+1+1			cf. PACS-CS 08
Laiho 11	[8]	2+1	2.5, 2.9, 3.0, 3.6, 3.8, 4.8	4.1 (val.) 4.1 (sea)	Data corrected using NLO $SU(3)$ χPT finite-V formulae.

Table 67: Finite volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PACS-CS 10	[11]	2+1			cf. PACS-CS 08
MILC 10A	[9]	2+1			cf. MILC 09A, 09
BMW 10A, 10B	[12, 13]	2+1	$\gtrsim 5.0$	$\gtrsim 4.0$	FV corrections below 5 per mil on the largest lattices.
RBC/UKQCD 10A	[14]	2+1	2.7	$\gtrsim 4.0$	
Blum 10	[15]	2+1	1.8, 2.7	—	Simulations done with quenched photons; large finite volume effects analytically corrected for, but not related to $M_\pi L$.
PACS-CS 09	[16]	2+1	2.9	2.0	Only one volume.
HPQCD 09A, 10	[17, 18]	2+1			cf. MILC 09
MILC 09A, 09	[19, 20]	2+1	2.5, 2.9, 3.4, 3.6, 3.8, 5.8	4.1, 3.8	
PACS-CS 08	[21]	2+1	2.9	2.3	Correction for FV from χPT using [43].
RBC/UKQCD 08	[22]	2+1	1.8, 2.7	4.6	Various volumes for comparison and correction for FV from χPT [43–45].
CP-PACS/JLQCD 07	[23]	2+1	2.0	6.0	Estimate based on the comparison to a $L = 1.6$ fm volume assuming powerlike dependence on L.
HPQCD 05	[24]	2+1	2.4, 2.9	3.5	
HPQCD/MILC/UKQCD 04, MILC 04	[25, 26]	2+1	2.4, 2.9	3.5	NLO SχPT.

Table 67: (cntd.) Finite volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 14D	[27]	2	2.2, 4.5	3.2	
RM123 13	[28]	2	$\gtrsim 2.0$	3.5	One volume $L = 1.7$ fm at $m_\pi = 495$, $a = 0.054$fm.
ALPHA 12	[29]	2	2.1–3.2	4.2	Roughly 2 distinct volumes; no analysis of FV effects.
RM123 11	[30]	2	$\gtrsim 2.0$	3.5	One volume $L = 1.7$ fm at $m_\pi = 495$, $a = 0.054$fm.
Dürr 11	[31]	2	1.22-2.30	2.8	A number of volumes in determination of m_c/m_s, but all but one have $L < 2$ fm.
ETM 10B	[32]	2	$\gtrsim 2.0$	3.5	One volume $L = 1.7$ fm at $m_\pi = 495$, $a = 0.054$fm.
JLQCD/TWQCD 08A	[33]	2	1.9	2.8	Corrections for FV based on NLO χPT.
RBC 07	[34]	2	1.9	4.3	Estimate of FV effect based on a model.
ETM 07	[35]	2	2.1	3.2	NLO PQχPT
QCDSF/UKQCD 06	[36]	2	1.4–1.9	4.7	
SPQcdR 05	[37]	2	1.0–1.5	4.3	Comparison between 1.0 and 1.5 fm.
ALPHA 05	[38]	2	2.6	7.4	
QCDSF/UKQCD 04	[39]	2	1.7–2.0	4.7	
JLQCD 02	[40]	2	1.8	5.1	Numerical study with three volumes.
CP-PACS 01	[41]	2	2.0–2.6	5.7	

Table 68: Finite volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	Description
HPQCD 14A	[1]	2+1+1	Renormalization not required through the use of the ratio m_c/m_s.
FNAL/MILC 14A	[2]	2+1+1	Renormalization not required for m_s/m_{ud}.
ETM 14	[3]	2+1+1	Non-perturbative renormalization (RI/MOM).

Table 69: Renormalization in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
QCDSF/UKQCD 15	[4]	2+1	Non-perturbative renormalization (RI/MOM).
RBC/UKQCD 14B	[5]	2+1	Non-perturbative renormalization (RI/SMOM).
RBC/UKQCD 12	[6]	2+1	Non-perturbative renormalization (RI/SMOM).
PACS-CS 12	[7]	1+1+1	cf. PACS-CS 10
Laiho 11	[8]	2+1	Z_A from AWI and Z_A/Z_S-1 from 1-loop, tadpole-improved, perturbation theory.
PACS-CS 10	[11]	2+1	Non-perturbative renormalization and running; Schrödinger functional method.
MILC 10A	[9]	2+1	cf. MILC 09A, 09
BMW 10A, 10B	[12, 13]	2+1	Non-perturbative renormalization (tree-level improved RI-MOM), nonperturbative running.
RBC/UKQCD 10A	[14]	2+1	Non-perturbative renormalization (RI/SMOM).
Blum 10	[15]	2+1	Relies on nonperturbative renormalization factors calculated by RBC/UKQCD 08; no QED renormalization.
PACS-CS 09	[16]	2+1	Non-perturbative renormalization; Schrödinger functional method.
HPQCD 09A, 10	[17, 18]	2+1	Lattice calculation of m_s/m_c: m_s derived from a perturbative determination of m_c.
MILC 09A, 09	[19, 20]	2+1	2-loop perturbative renormalization.
PACS-CS 08	[21]	2+1	1-loop perturbative renormalization.
RBC/UKQCD 08	[22]	2+1	Non-perturbative renormalization, 3-loop perturbative matching.
CP-PACS/JLQCD 07	[23]	2+1	1-loop perturbative renormalization, tadpole improved.
HPQCD 05	[24]	2+1	2-loop perturbative renormalization.
HPQCD/MILC/UKQCD 04, MILC 04	[25, 26]	2+1	1-loop perturbative renormalization.

Table 70: Renormalization in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	Description
ETM 14D	[27]	2	Renormalization not required for m_s/m_{ud}.
RM123 13	[28]	2	Non-perturbative renormalization.
ALPHA 12	[29]	2	Non-perturbative renormalization.
RM123 11	[30]	2	Non-perturbative renormalization.
Dürr 11	[31]	2	Lattice calculation of m_s/m_c: m_s derived from a perturbative determination of m_c.
ETM 10B	[32]	2	Non-perturbative renormalization.
JLQCD/TWQCD 08A	[33]	2	Non-perturbative renormalization.
RBC 07	[34]	2	Non-perturbative renormalization.
ETM 07	[35]	2	Non-perturbative renormalization.
QCDSF/UKQCD 06	[36]	2	Non-perturbative renormalization.
SPQcdR 05	[37]	2	Non-perturbative renormalization.
ALPHA 05	[38]	2	Non-perturbative renormalization.
QCDSF/UKQCD 04	[39]	2	Non-perturbative renormalization.
JLQCD 02	[40]	2	1-loop perturbative renormalization.
CP-PACS 01	[41]	2	1-loop perturbative renormalization.

Table 71: Renormalization in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
HPQCD 14A	[1]	2+1+1	0.06, 0.09, 0.12, 0.15	Scale set through the Wilson flow parameter w_0.
ETM 14	[3]	2+1+1	0.062, 0.082, 0.089	Scale set through F_π.
ETM 14A	[46]	2+1+1	0.062, 0.082, 0.089	Scale set through the nucleon mass M_N.
JLQCD 15B	[47]	2+1	0.044, 0.055. 0.083	Möbius domain wall fermions.
χQCD 14	[48]	2+1	0.087, 0.11	Overlap valence fermions on domain-wall sea quarks from [14]. The lattice scale is set together with the strange and charm quark masses using the experimental values of the D_s, D_s^* and J/ψ meson masses.
HPQCD 10	[18]	2+1	0.044, 0.059, 0.085, 0.12, 0.15	Scale set through the static-quark potential parameter r_1.
HPQCD 08B	[49]	2+1	0.06, 0.09, 0.12, 0.15	Scale set through the static-quark potential parameter r_1.
ALPHA 13B	[50]	2	0.048, 0.065	Scale set through F_K.
ETM 11F	[51]	2		cf. ETM 10B
ETM 10B	[32]	2	0.054, 0.067, 0.085, 0.098	Scale set through F_π.

Table 72: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_c.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
HPQCD 14A	[1]	2+1+1	$128_{\pi,5}$ (173_{RMS})	
ETM 14	[3]	2+1+1	180_{π^0} $(220_{\pi^\pm})$	
ETM 14A	[46]	2+1+1	210	cf. ETM 14
JLQCD 15B	[47]	2+1		
χQCD 14	[48]	2+1	290	
HPQCD 10	[18]	2+1	260	
HPQCD 08B	[49]	2+1		
ALPHA 13B	[50]	2	190	
ETM 11F	[51]	2		cf. ETM 10B
ETM 10B	[32]	2	270	

Table 73: Chiral extrapolation/minimum pion mass in the determinations of m_c.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
HPQCD 14A	[1]	2+1+1	2.5 - 5.8	3.7	
ETM 14	[3]	2+1+1	2.0 - 3.0	2.7_{π^0} $(3.3_{\pi^\pm})$	
ETM 14A	[46]	2+1+1	2.0 - 3.0	2.7_{π^0} $(3.3_{\pi^\pm})$	
JLQCD 15B	[47]	2+1	2.7		
χQCD 14	[48]	2+1	2.8	4.1	
HPQCD 10	[18]	2+1	2.3 - 3.4	3.8	
HPQCD 08B	[49]	2+1			
ALPHA 13B	[50]	2	4.2	4.0	
ETM 11F	[51]	2			cf. ETM 10B
ETM 10B	[32]	2	$\gtrsim 2.0$	3.5	

Table 74: Finite volume effects in the determinations of m_c.

Collab.	Ref.	N_f	Description
HPQCD 14A	[1]	2+1+1	Renormalization not required.
ETM 14	[3]	2+1+1	Non-perturbative renormalization (RI/MOM).
ETM 14A	[46]	2+1+1	Non-perturbative renormalization (RI/MOM).
JLQCD 15B	[47]	2+1	Renormalization not required.
χQCD 14	[48]	2+1	Non-perturbative renormalization (RI/MOM).
HPQCD 10	[18]	2+1	Renormalization not required.
HPQCD 08B	[49]	2+1	Renormalization not required.
ALPHA 13B	[50]	2	Non-perturbative renormalization (RI/MOM) plus 1-loop PT estimate for the improvement b-coefficients.
ETM 11F	[51]	2	Renormalization not required.
ETM 10B	[32]	2	Non-perturbative renormalization (RI/MOM).

Table 75: Renormalization in the determinations of m_c.

Collab.	Ref.	N_f	a [fm]	Description
HPQCD 14B	[52]	2+1+1	0.09, 0.12, 0.15	Scale set through the $\Upsilon' - \Upsilon$ mass splitting.
ETM 14B	[53]	2+1+1	0.062, 0.082, 0.089	Scale set through F_π.
HPQCD 14A	[1]	2+1+1	0.06, 0.09, 0.12, 0.15	Scale set through the Wilson flow parameter w_0.
HPQCD 13B	[54]	2+1	0.084, 0.12	Scale set through the static-quark potential parameter r_1.
HPQCD 10	[18]	2+1	0.044, 0.059, 0.084, 0.12, 0.15	Scale set through the static-quark potential parameter r_1.
ETM 13B	[55]	2	0.054, 0.067, 0.085, 0.098	Scale set through the static-quark potential parameter r_0.
ALPHA 13C	[56]	2	0.048, 0.065, 0.075	Scale set through F_K.
ETM 11A	[57]	2	0.054, 0.067, 0.085, 0.098	Scale set through F_π.

Table 76: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_b.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]
HPQCD 14B	[52]	2+1+1	306, 128
ETM 14B	[53]	2+1+1	210
HPQCD 14A	[1]	2+1+1	$128_{\pi,5}$ (173_{RMS})
HPQCD 13B	[54]	2+1	345
HPQCD 10	[18]	2+1	260
ETM 13B	[55]	2	280
ALPHA 13C	[56]	2	190
ETM 11A	[57]	2	280

Table 77: Chiral extrapolation/minimum pion mass in the determinations of m_b.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$
HPQCD 14B	[52]	2+1+1	2.4-7.8	3.0-3.8
ETM 14B	[53]	2+1+1	1.9-2.8	3.0-5.8
HPQCD 14A	[1]	2+1+1	2.5-5.8	3.7
HPQCD 13B	[54]	2+1	2.4, 3.4	4.1
HPQCD 10	[18]	2+1	2.3 - 3.4	3.8
ETM 13B	[55]	2	$\gtrsim$ 2.0	3.5
ALPHA 13C	[56]	2	2.3-3.6	4.1
ETM 11A	[57]	2	$\gtrsim$ 2.0	3.5

Table 78: Finite volume effects in the determinations of m_b.

Collab.	Ref.	N_f	Description
HPQCD 14B	[52]	2+1+1	Renormalization not required.
ETM 14B	[53]	2+1+1	Non-perturbative renormalization (RI/MOM).
HPQCD 14A	[1]	2+1+1	Renormalization not required.
HPQCD 13B	[54]	2+1	Renormalization not required.
HPQCD 10	[18]	2+1	Renormalization not required.
ETM 13B	[55]	2	Non-perturbative renormalization (RI/MOM).
ALPHA 13C	[56]	2	Non-perturbatively matched and renormalized HQET.
ETM 11A	[57]	2	Renormalization not required.

Table 79: Lattice renormalization in the determinations of m_b.

B.2 Notes to section 4 on $|V_{ud}|$ and $|V_{us}|$

Collab.	Ref.	N_f	a [fm]	Description
ETM 16	[58]	2+1+1	0.062, 0.082, 0.089	Scale set through f_π. Automatic $\mathcal{O}(a)$ improvement.
FNAL/MILC 13E	[59]	2+1+1	0.06, 0.09, 0.12, 0.15	HISQ action for both sea and valence quarks. Relative scale through r_1, physical scale from pseudoscalar decay constants calculated with Asqtad fermions. The ensemble with $a \simeq 0.06$ fm is used only for cross-checking discretization effects.
FNAL/MILC 13C	[60]	2+1+1	0.09, 0.12, 0.15	Relative scale through r_1, physical scale from f_π calculated by MILC 09A at $N_f = 2+1$.
JLQCD 16	[61]	2+1	0.112	Scale set through Ω mass.
RBC/UKQCD 15A	[62]	2+1	0.08, 0.11	Scale set through Ω mass.
FNAL/MILC 12I	[63]	2+1	0.09, 0.12	Relative scale r_1, physical scale determined from a mixture of f_π, f_K, radial excitation of Υ and $m_{D_s} - \frac{1}{2}m_{\eta_c}$.
RBC/UKQCD 13	[64]	2+1	0.09, 0.11, 0.14	Scale set through Ω mass.
JLQCD 12	[65]	2+1	0.112	Scale set through Ω mass.
JLQCD 11	[66]	2+1	0.112	Scale set through Ω mass.
RBC/UKQCD 07,10	[67, 68]	2+1	0.114(2)	Scale fixed through Ω baryon mass. Add $(\Lambda_{\rm QCD}a)^2 \approx 4\%$ systematic error for lattice artifacts. Fifth dimension with extension $L_s = 16$, therefore small residual chiral symmetry breaking and approximate $\mathcal{O}(a)$-improvement.
ETM 10D	[69]	2	0.05, 0.07, 0.09, 0.10	Scale set through f_π. Automatic $\mathcal{O}(a)$ impr., flavour symmetry breaking: $(M^0_{PS})^2 - (M^\pm_{PS})^2 \sim \mathcal{O}(a^2)$.
ETM 09A	[70]	2	0.07, 0.09, 0.10	Scale set through f_π. Automatic $\mathcal{O}(a)$ impr., flavour symmetry breaking: $(M^0_{PS})^2 - (M^\pm_{PS})^2 \sim \mathcal{O}(a^2)$. Three lattice spacings only for pion mass 470 MeV.
QCDSF 07	[71]	2	0.075	Scale set with r_0. Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions, not clear whether currents improved.
RBC 06	[72]	2	0.12	Scale set through M_ρ. Automatic $\mathcal{O}(a)$-improvement due to approximate chiral symmetry of the action.
JLQCD 05	[73]	2	0.0887	Scale set through M_ρ. Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions.

Table 80: Continuum extrapolations/estimation of lattice artifacts in the determinations of $f_+(0)$.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 16	[58]	2+1+1	$180_{\pi^0}(220_{\pi^\pm})$	Chiral extrapolation performed through $SU(2)$ or $SU(3)$ χPT.
FNAL/MILC 13E	[59]	2+1+1	$173_{\rm RMS}(128_{\pi,5})$	NLO $SU(3)$ PQ staggered χPT with continuum χPT at NNLO. Lightest Nambu-Goldstone mass is 128 MeV and lightest RMS mass is 173 MeV for the same gauge ensemble with $a \simeq 0.09$ fm.
FNAL/MILC 13C	[60]	2+1+1	$173_{\rm RMS}(128_{\pi,5})$	NLO $SU(3)$ PQ staggered χPT with continuum χPT at NNLO. Lightest Nambu-Goldstone mass is 128 MeV and lightest RMS mass is 173 MeV for the same gauge ensemble with $a \simeq 0.09$ fm.
JLQCD 16	[61]	2+1	290	NNLO $SU(3)$ χPT.
RBC/UKQCD 15A	[62]	2+1	140	NLO $SU(3)$ χPT with phenomenological ansatz for higher orders or polynomial models.
FNAL/MILC 12I	[63]	2+1	$378_{\rm RMS}(263_{\pi,5})$	NLO $SU(3)$ PQ staggered χPT with either phenomenological NNLO ansatz or NNLO χPT. Lightest Nambu-Goldstone mass is 263 MeV with $a = 0.12$ fm and lightest RMS mass is 378 MeV with $a = 0.09$ fm.
RBC/UKQCD 13	[64]	2+1	170	NLO $SU(3)$ χPT with phenomenological ansatz for higher orders.
JLQCD 12	[65]	2+1	290	NLO $SU(3)$ χPT with phenomenological ansatz for higher orders.
JLQCD 11	[66]	2+1	290	NLO $SU(3)$ χPT with phenomenological ansatz for higher orders.
RBC/UKQCD 07,10	[67, 68]	2+1	330	NLO $SU(3)$ χPT with phenomenological ansatz for higher orders.
ETM 10D	[69]	2	$210_{\pi^0}(260_{\pi^\pm})$	NLO heavy kaon $SU(2)$ χPT and NLO $SU(3)$ χPT and phenomenological ansatz for higher orders. Average of $f_+(0)$-fit and joint $f_+(0)$-f_K/f_π-fit.
ETM 09A	[70]	2	$210_{\pi^0}(260_{\pi^\pm})$	NLO heavy kaon $SU(2)$ χPT and NLO $SU(3)$ χPT and phenomenological ansatz for higher orders.
QCDSF 07	[71]	2	591	Only one value for the pion mass.
RBC 06	[72]	2	490	NLO $SU(3)$ χPT and phenomenological ansatz for higher orders.
JLQCD 05	[73]	2	550	NLO $SU(3)$ χPT and phenomenological ansatz for higher orders.

Table 81: Chiral extrapolation/minimum pion mass in determinations of $f_+(0)$. The subscripts RMS and $\pi,5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass, respectively. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass where applicable.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
ETM 16	[58]	2+1+1	2.0–3.0	$2.7_{\pi^0}(3.3_{\pi^\pm})$	FSE observed only in the slopes of the vector and scalar form factors.
FNAL/MILC 13E	[59]	2+1+1	2.9–5.8	$4.9_{\rm RMS}(3.6_{\pi,5})$	The values correspond to $M_{\pi,\rm RMS}$ = 173 MeV and $M_{\pi,5} = 128$ MeV, respectively.
FNAL/MILC 13C	[60]	2+1+1	2.9–5.8	$4.9_{\rm RMS}(3.6_{\pi,5})$	The values correspond to $M_{\pi,\rm RMS}$ = 173 MeV and $M_{\pi,5} = 128$ MeV, respectively.
JLQCD 16	[61]	2+1	1.8, 2.7	4.1	
RBC/UKQCD 15A	[62]	2+1	2.6, 5.2	3.9	
FNAL/MILC 12I	[63]	2+1	2.4–3.4	$6.2_{\rm RMS}(3.8_{\pi,5})$	The values correspond to $M_{\pi,\rm RMS}$ = 378 MeV and $M_{\pi,5} = 263$ MeV, respectively.
RBC/UKQCD 13	[64]	2+1	2.7, 4.6	3.9	
JLQCD 12	[65]	2+1	1.8, 2.7	4.1	
JLQCD 11	[66]	2+1	1.8, 2.7	4.1	
RBC/UKQCD 07,10	[67, 68]	2+1	1.8,2.7	4.7	Two volumes for all but the lightest pion mass.
ETM 10D	[69]	2	2.1–2.8	$3.0_{\pi^0}(3.7_{\pi^\pm})$	
ETM 09A	[70]	2	2.1, 2.8	$3.0_{\pi^0}(3.7_{\pi^\pm})$	Two volumes at $M_\pi = 300$ MeV and χPT-motivated estimate of the error due to FSE.
QCDSF 07	[71]	2	1.9	5.4	
RBC 06	[72]	2	1.9	4.7	
JLQCD 05	[73]	2	1.8	4.9	

Table 82: Finite volume effects in determinations of $f_+(0)$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass, respectively. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass where applicable.

Collab.	Ref.	N_f	a [fm]	Description
ETM 14E	[74]	2+1+1	0.062, 0.082, 0.089	Scale set through f_π. Automatic $\mathcal{O}(a)$ improvement, flavour symmetry breaking: $(M_{PS}^0)^2 - (M_{PS}^\pm)^2 \sim \mathcal{O}(a^2)$. Discretization and volume effects due to the $\pi^0 - \pi^\pm$ mass splitting are taken into account through χPT for twisted-mass fermions.
FNAL/MILC 14A	[2]	2+1+1	0.06, 0.09, 0.12, 0.15	HISQ action for both valence and sea quarks. Absolute scale though f_π.
HPQCD 13A	[75]	2+1+1	0.09, 0.12, 0.15	Relative scale through Wilson flow and absolute scale through f_π.
MILC 13A	[76]	2+1+1	0.06, 0.09, 0.12, 0.15	Absolute scale though f_π.
ETM 13F	[77]	2+1+1	0.062, 0.082, 0.089	Scale set through f_π. Automatic $\mathcal{O}(a)$ improvement, flavour symmetry breaking: $(M_{PS}^0)^2 - (M_{PS}^\pm)^2 \sim \mathcal{O}(a^2)$. Discretization and volume effects due to the $\pi^0 - \pi^\pm$ mass splitting are taken into account through χPT for twisted-mass fermions.
ETM 10E	[78]	2+1+1	0.061, 0.078	Scale set through f_π/m_π. Two lattice spacings but a-dependence ignored in all fits. Finer lattice spacing from [79].
MILC 11	[80]	2+1+1	0.09, 0.12	Relative scale through $f_{\rm PS}/m_{\rm PS}$ = fixed, absolute scale though f_π.

Table 83: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π for $N_f = 2+1+1$ simulations.

Collab.	Ref.	N_f	a [fm]	Description
BMW 16	[81, 82]	2+1	0.054, 0.065, 0.077, 0.092, 0.12	Scale set through M_Ω. Perturbative $\mathcal{O}(a)$-improvement.
RBC/UKQCD 14B	[5]	2+1	0.063, 0.085, 0.114	Scale set through m_Ω.
RBC/UKQCD 12	[6]	2+1	0.09, 0.11, 0.14	Scale set through m_Ω.
Laiho 11	[8]	2+1	0.06, 0.09, 0.125	Scale set through r_1 and Υ and continuum extrapolation based on MAχPT.
JLQCD/TWQCD 10	[83]	2+1	0.112	Scale set through M_Ω.
RBC/UKQCD 10A	[14]	2+1	0.087, 0.114	Scale set through M_Ω.
MILC 10	[84]	2+1	0.045, 0.06, 0.09	3 lattice spacings, continuum extrapolation by means of RSχPT.
BMW 10	[85]	2+1	0.07, 0.08, 0.12	Scale set through $M_{\Omega,\Xi}$. Perturbative $\mathcal{O}(a)$-improvement.
JLQCD/TWQCD 09A	[33]	2+1	0.1184(3)(21)	Scale set through F_π. Automatic $\mathcal{O}(a)$-improvement due to chiral symmetry of action.
PACS-CS 09	[16]	2+1	0.0900(4)	Scale set through M_Ω.
MILC 09A	[19]	2+1	0.045, 0.06, 0.09	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT.
MILC 09	[20]	2+1	0.045, 0.06, 0.09, 0.12	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT.
Aubin 08	[86]	2+1	0.09, 0.12	Scale set through r_1 and Υ and continuum extrapolation based on MAχPT.
PACS-CS 08, 08A	[21, 87]	2+1	0.0907(13)	Scale set through M_Ω. Non-perturbatively $\mathcal{O}(a)$-improved.
HPQCD/UKQCD 07	[88]	2+1	0.09, 0.12, 0.15	Scale set through r_1 and Υ and continuum extrapolation on continuum-χPT motivated ansatz. Taste breaking of sea quarks ignored.
RBC/UKQCD 08	[22]	2+1	0.114(2)	Scale set through M_Ω. Automatic $\mathcal{O}(a)$-improvement due to appoximate chiral symmetry. $(\Lambda_{\rm QCD}a)^2 \approx 4\%$ systematic error due to lattice artifacts added.
NPLQCD 06	[89]	2+1	0.125	Scale set through r_0 and F_π. Taste breaking of sea quarks ignored.
MILC 04	[26]	2+1	0.09, 0.12	Scale set through r_1 and Υ and continuum extrapolation based on RSχPT.

Table 84: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π for $N_f = 2+1$ simulations.

Collab.	Ref.	N_f	a [fm]	Description
ETM 14D	[27]	2	0.094	Scale set through F_π, r_0, t_0 and w_0. Twisted Wilson fermions plus clover term. Automatic $\mathcal{O}(a)$ improvement.
ALPHA 13A	[90]	2	0.05, 0.065, 0.075	Scale set through F_π. $\mathcal{O}(a)$-improved Wilson action.
BGR 11	[91]	2	0.135	Scale set through $r_0 = 0.48$ fm. Chirally improved Dirac operator.
ETM 10D	[69]	2	0.05, 0.07, 0.09, 0.10	Scale set through F_π. Automatic $\mathcal{O}(a)$ impr., flavour symmetry breaking: $(M^0_{PS})^2 - (M^\pm_{PS})^2 \sim \mathcal{O}(a^2)$.
ETM 09	[92]	2	0.07, 0.09, 0.10	Scale set through F_π. Automatic $\mathcal{O}(a)$ impr., flavour symmetry breaking: $(M^0_{PS})^2 - (M^\pm_{PS})^2 \sim \mathcal{O}(a^2)$.
QCDSF/UKQCD 07	[93]	2	0.06, 0.07	Scale set through F_π. Non-perturbative $\mathcal{O}(a)$-improvement.

Table 85: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π for $N_f = 2$ simulations.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
ETM 14E	[74]	2+1+1	$180_{\pi^0}(220_{\pi^\pm})$	Chiral extrapolation performed through $SU(2)$ χPT or polynomial fit.
FNAL/MILC 14A	[2]	2+1+1	$143_{\mathrm{RMS}}(128_{\pi,5})$	Linear interpolation to physical point. The lightest RMS mass is from the $a = 0.06$ fm ensemble and the lightest Nambu-Goldstone mass is from the $a = 0.09$ fm ensemble.
HPQCD 13A	[75]	2+1+1	$173_{\mathrm{RMS}}(128_{\pi,5})$	NLO χPT supplemented by model for NNLO. Both the lightest RMS and the lightest Nambu-Goldstone mass are from the $a = 0.09$ fm ensemble.
MILC 13A	[76]	2+1+1	$143_{\mathrm{RMS}}(128_{\pi,5})$	Linear interpolation to physical point. The lightest RMS mass is from the $a = 0.06$ fm ensemble and the lightest Nambu-Goldstone mass is from the $a = 0.09$ fm ensemble.
ETM 13F	[77]	2+1+1	$180_{\pi^0}(220_{\pi^\pm})$	Chiral extrapolation performed through $SU(2)$ χPT or polynomial fit.
ETM 10E	[78]	2+1+1	$215_{\pi^0}(265_{\pi^\pm})$	
MILC 11	[80]	2+1+1	$173_{\mathrm{RMS}}(128_{\pi,5})$	Quoted result from polynomial interpolation to the physical point. The lightest RMS mass is from the $a = 0.06$ fm ensemble and lightest the Nambu-Goldstone mass is from the $a = 0.09$ fm ensemble.

Table 86: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2+1+1$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and, where applicable, "val" and "sea" indicate valence and sea pion masses.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
BMW 16	[81, 82]	2+1	130	Comparison between $SU(3)$ χPT and polynomial it-ansätze.
RBC/UKQCD 14B	[5]	2+1	139	NLO PQ $SU(2)$ χPT as well as analytic ansätze.
RBC/UKQCD 12	[6]	2+1	171_{sea}, 143_{val}	NLO PQ $SU(2)$ χPT as well as analytic ansätze.
Laiho 11	[8]	2+1	$250_{\text{RMS}}(220_{\pi,5})$	NLO MAχPT.
JLQCD/TWQCD 10	[83]	2+1	290	NNLO χPT.
RBC/UKQCD 10A	[14]	2+1	290	Results are based on heavy kaon NLO $SU(2)$ PQχPT.
MILC 10	[84]	2+1	$258_{\text{RMS}}(177_{\pi,5})$	Lightest Nambu-Goldstone mass is 177 MeV (at 0.09 fm) and lightest RMS mass is 258 MeV (at 0.06 fm). NLO rSχPT and NNLO χPT.
BMW 10	[85]	2+1	190	Comparison of various fit-ansätze: $SU(3)$ χPT, heavy kaon $SU(2)$ χPT, polynomial.
JLQCD/TWQCD 09A	[33]	2+1	290	NNLO $SU(3)$ χPT.
PACS-CS 09	[16]	2+1	156	NNLO χPT.
MILC 09A	[19]	2+1	$258_{\text{RMS}}(177_{\pi,5})$	NLO $SU(3)$ RSχPT, continuum χPT at NNLO and up to NNNNLO analytic terms. Heavy kaon $SU(2)$ RSχPT with NNLO continuum chiral logs on a sub-set of the lattices. The lightest Nambu-Goldstone mass is 177 MeV (at $a = 0.09$ fm) and the lightest RMS mass is 258 MeV (at $a = 0.06$ fm).
MILC 09	[20]	2+1	$258_{\text{RMS}}(224_{\pi,5})$	NLO $SU(3)$ RSχPT with continuum χPT NNLO and NNNLO analytic terms added. According to [19] the lightest sea Nambu-Goldstone mass is 224 MeV and the lightest RMS mass is 258 MeV (at $a = 0.06$ fm).

Table 87: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2+1$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, “val” and “sea” indicate valence and sea pion masses.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
Aubin 08	[86]	2+1	$329_{\text{RMS}}(246_{\pi,5})$	NLO MAχPT. According to [19] the lightest sea Nambu-Goldstone mass is 246 MeV (at $a = 0.09$ fm) and the lightest RMS mass is 329 MeV (at $a = 0.09$ fm).
PACS-CS 08, 08A	[21, 87]	2+1	156	NLO $SU(2)$ χPT and $SU(3)$ (Wilson)χPT.
HPQCD/UKQCD 07	[88]	2+1	$375_{\text{RMS}}(263_{\pi,5})$	NLO $SU(3)$ chiral perturbation theory with NNLO and NNNLO analytic terms. The lightest RMS mass is from the $a = 0.09$ fm ensemble and the lightest Nambu-Goldstone mass is from the $a = 0.12$ fm ensemble.
RBC/UKQCD 08	[22]	2+1	330_{sea}, 242_{val}	While $SU(3)$ PQχPT fits were studied, final results are based on heavy kaon NLO $SU(2)$ PQχPT.
NPLQCD 06	[89]	2+1	300	NLO $SU(3)$ χPT and some NNLO terms. The sea RMS mass for the employed lattices is heavier.
MILC 04	[26]	2+1	$400_{\text{RMS}}(260_{\pi,5})$	PQ RSχPT fits. The lightest sea Nambu-Goldstone mass is 260 MeV (at $a = 0.12$ fm) and the lightest RMS mass is 400 MeV (at $a = 0.09$ fm).

Table 87: (cntd.) Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2+1$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, "val" and "sea" indicate valence and sea pion masses.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ETM 14D	[27]	2	140	Charged/neutral pion mass breaking, $M^2_{\pi^\pm} - M^2_{\pi^0} \sim \mathcal{O}(a^2)$, estimated to be $\simeq 20$ MeV.
ALPHA 13A	[90]	2	190	NLO $SU(3)$ χPT and phenomenological ansatz for higher orders.
BGR 11	[91]	2	250	NLO $SU(2)$ χPT. Strange quark mass fixed by reproducing the Ω mass.
ETM 10D	[69]	2	$210_{\pi^0}(260_{\pi^\pm})$	NLO $SU(3)$ χPT and phenomenological ansatz for higher orders. Joint $f_+(0)$-f_K/f_π-fit.
ETM 09	[92]	2	$210_{\pi^0}(260_{pi^\pm})$	NLO heavy meson $SU(2)$ χPT and NLO $SU(3)$ χPT.
QCDSF/UKQCD 07	[93]	2	300	Linear extrapolation of lattice data.

Table 88: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, "val" and "sea" indicate valence and sea pion masses.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 14E	[74]	2+1+1	2.0 - 3.0	$2.7_{\pi^0}(3.3_{\pi^\pm})$	FSE for the pion is corrected through resummed NNLO χPT for twisted-mass fermions, which takes into account the effects due to the $\pi^0 - \pi^\pm$ mass splitting.
FNAL/MILC 14A	[2]	2+1+1	2.8-5.8	$3.9_{\rm RMS}(3.7_{\pi,5})$	
HPQCD 13A	[75]	2+1+1	2.5-5.8	$4.9_{\rm RMS}(3.7_{\pi,5})$	
MILC 13A	[76]	2+1+1	2.8-5.8	$3.9_{\rm RMS}(3.7_{\pi,5})$	
ETM 13F	[77]	2+1+1	2.0 - 3.0	$2.7_{\pi^0}(3.3_{\pi^\pm})$	FSE for the pion is corrected through resummed NNLO χPT for twisted-mass fermions, which takes into account the effects due to the $\pi^0 - \pi^\pm$ mass splitting.
ETM 10E	[78]	2+1+1	1.9 - 2.9	$3.1_{\pi^0}(3.9_{\pi^\pm})$	Simulation parameters from [79, 94].
MILC 11	[80]	2+1+1	5.6, 5.7	$4.9_{\rm RMS}(3.7_{\pi,5})$	

Table 89: Finite volume effects in determinations of f_K/f_π for $N_f = 2+1+1$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, “val” and “sea” indicate valence and sea pion masses.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
BMW 16	[81, 82]	2+1	1.5–5.5	3.85	Various volumes for comparison and corrections for FSE from NLO χPT with re-fitted coefficients.
RBC/UKQCD 14B	[5]	2+1	$2.0, 2.7, 4.6, 5.4$	3.8	
RBC/UKQCD 12	[6]	2+1	$2.7, 4.6$	3.3	For partially quenched M_π = 143MeV, $M_\pi L = 3.3$ and for unitary $M_\pi = 171$MeV, $M_\pi L = 4.0$.
Laiho 11	[8]	2+1	2.5–4.0	$4.9_{\rm RMS}(4.3_{\pi,5})$	
JLQCD/TWQCD 10	[83]	2+1	1.8, 2.7	4.0	
RBC/UKQCD 10A	[14]	2+1	2.7	4.0	$M_\pi L = 4.0$ for lightest sea quark mass and $M_\pi L = 3.1$ for lightest partially quenched quark mass.
MILC 10	[84]	2+1	2.5-3.8	$7.0_{\rm RMS}(4.0_{\pi,5})$	$L \geq 2.9$ fm for the lighter masses.
BMW 10	[85]	2+1	2.0–5.3	4.0	Various volumes for comparison and correction for FSE from χPT using [43].
JLQCD/TWQCD 09A	[33]	2+1	1.9	2.8	Estimate of FSE using χPT [43, 95].
PACS-CS 09	[16]	2+1	2.9	2.28	After reweighting to the physical point $M_{\pi,\min}L = 1.97$.
MILC 09A	[19]	2+1	2.5–5.8	$7.0_{\rm RMS}(4.1_{\pi,5})$	
MILC 09	[20]	2+1	2.4–5.8	$7.0_{\rm RMS}(4.8_{\pi,5})$	Various volumes for comparison and correction for FSEs from (RS)χPT [43].
Aubin 08	[86]	2+1	2.4–3.6	4.0	Correction for FSE from MAχPT.
PACS-CS 08, 08A	[21, 87]	2+1	2.9	2.3	Correction for FSE from χPT using [43].
HPQCD/UKQCD 07	[88]	2+1	2.4–2.9	$4.1_{\rm RMS}(3.8_{\pi,5})$	Correction for FSE from χPT using [43].
RBC/UKQCD 08	[22]	2+1	1.8, 2.7	$4.6_{\rm sea}$, 3.4_{rval}	Various volumes for comparison and correction for FSE from χPT [43–45].
NPLQCD 06	[89]	2+1	2.5	3.8	Correction for FSE from SχPT [96, 97].
MILC 04	[26]	2+1	2.4, 3.0	$4.8_{\rm RMS}(3.8_{\pi,5})$	NLO SχPT.
ETM 14D	[27]	2	$2.2, 4.5$	3.2	
ALPHA 13A	[90]	2	2.1, 2.4, 3.1	4.0	
BGR 11	[91]	2	2.1, 2.2	2.7	
ETM 10D	[69]	2	2.1–2.8	$3.0_{\pi^0}(3.7_{\pi^\pm})$	
ETM 09	[92]	2	2.0–2.7	$3.0_{\pi^0}(3.7_{\pi^\pm})$	Correction for FSE from χPT [43–45].
QCDSF/UKQCD 07	[93]	2	1.4,...,2.6	4.2	Correction for FSE from χPT.

Table 90: Finite volume effects in determinations of f_K/f_π for $N_f = 2+1$ and $N_f = 2$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, "val" and "sea" indicate valence and sea pion masses.

B.3 Notes to section 5 on Low-Energy Constants

Collab.	Ref.	N_f	a [fm]	Description
HPQCD 13A, 15B	[75, 98]	2+1+1	0.09–0.15	Configurations are shared with MILC.
ETM 11, 13	[79, 99]	2+1+1	0.0607–0.0863	3 lattice spacings fixed through F_π/M_π.
ETM 10	[100]	2+1+1	0.078, 0.086	Fixed through F_π/M_π.
JLQCD 16B	[37]	2+1	0.04–0.08	Fixed through $\sqrt{t_0}$ = 0.1465(21)(13) fm.
JLQCD 15A	[102]	2+1	0.112	Fixed through Ω baryon mass
RBC/UKQCD 14B, 15E	[5, 103]	2+1	a^{-1} =1.730–3.148	Fixed through m_π, m_K, and m_Ω.
Boyle 14	[104]	2+1	a^{-1} =1.37, 2.31	Shared with RBC/UKQCD 12.
BMW 13	[105]	2+1	0.054–0.093	Scale set through Ω baryon mass.
RBC/UKQCD 12	[6]	2+1	0.086, 0.114 and 0.144 for $M_\pi^{m_{\rm R} min}$	Scale set through m_Ω.
Borsanyi 12	[106]	2+1	0.097–0.284	Scale fixed through F_π/M_π.
NPLQCD 11	[107]	2+1	0.09, 0.125	Configurations are shared with MILC 09 [20].
MILC 09, 09A, 10, 10A	[9, 19, 20, 84]	2+1	0.045–0.18	3 lattice spacings, continuum extrapolation by means of RSχPT.
JLQCD(/TWQCD) 08B, 09, 10A, 14	[108–111]	2+1, 3	0.11	One lattice spacing, fixed through m_Ω.
RBC/UKQCD 09, 10A	[14, 112]	2+1	0.1106(27), 0.0888(12)	Two lattice spacings. Data combined in global chiral-continuum fits.
TWQCD 08	[113]	2+1	0.122(3)	Scale fixed through m_ρ, r_0.
PACS-CS 08, 11A	[21, 114]	2+1	0.0907	One lattice spacing.
RBC/UKQCD 08A, 08	[22, 115]	2+1	0.114	One lattice spacing, attempt to estimate cut-off effects via formal argument.
NPLQCD 06	[89]	2+1	0.125	One lattice spacing, continuum χPT used.
LHP 04	[116]	2+1	$\simeq$ 0.12	Only one lattice spacing, mixed discretization approach.

Table 91: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1+1$ and $2+1$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	a [fm]	Description
ETMC 15A	[117]	2	0.0914(3)(15)	Weighted average using m_π, f_π, f_K, m_N.
Gülpers 15	[118]	2	0.050, 0.063, 0.079	Scale fixed through m_Ω.
Engel 14	[119]	2	0.0483(4), 0.0652(6), 0.0749(8)	Scale fixed through F_K.
Gülpers 13	[120]	2	0.063	Scale fixed through m_Ω.
Brandt 13	[121]	2	0.05–0.08	Configurations are shared with CLS.
QCDSF 13	[122]	2	0.06–0.076	Scale fixed through $r_0 = 0.50(1)$ fm.
Bernardoni 11	[123]	2	0.0649(10)	Configurations are shared with CLS.
TWQCD 11A, 11	[124, 125]	2	0.1034(1)(2)	Scale fixed through r_0.
Bernardoni 10	[126]	2	0.0784(10)	Scale fixed through M_K. Non-perturbative $\mathcal{O}(a)$ improvement. No estimate of systematic error.
ETM 09B	[127]	2	0.063, 0.073	Automatic $\mathcal{O}(a)$ impr. $r_0 = 0.49$ fm used.
ETM 09C, 12, 13	[99, 128, 129]	2	0.051–0.1	Automatic $\mathcal{O}(a)$ impr. Scale fixed through F_π. 4 lattice spacings, continuum extrapolation.
ETM 08	[130]	2	0.07-0.09	Automatic $\mathcal{O}(a)$ impr. Two lattice spacings. Scale fixed through F_π.
JLQCD/TWQCD 07, 07A, 08A, 09, 10A JLQCD 08A	[33, 110, 131–133], [134]	2	0.1184(3)(21)	Automatic $\mathcal{O}(a)$ impr., exact chiral symmetry. Scale fixed through r_0.
CERN 08	[135]	2	0.0784(10)	Scale fixed through M_K. Non-perturbative $\mathcal{O}(a)$ improvement.
Hasenfratz 08	[136]	2	0.1153(5)	Tree level $\mathcal{O}(a)$ improvement. Scale fixed through r_0. Estimate of lattice artifacts via WχPT [137].
CERN-TOV 06	[138]	2	0.0717(15), 0.0521(7), 0.0784(10)	Scale fixed through M_K. The lattice with $a = 0.0784(10)$ is obtained with non-perturbative $\mathcal{O}(a)$ improvement.
QCDSF/UKQCD 06A	[139]	2	0.07-0.115	5 lattice spacings. Non-perturbative $\mathcal{O}(a)$ improvement. Scale fixed through r_0.

Table 92: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
HPQCD 15B	[98]	2+1+1	175	Simulated at physical point.
HPQCD 13A	[75]	2+1+1	175	NLO chiral fit.
ETM 13	[99]	2+1+1	270	Linear fit in the quark mass.
ETM 11	[79]	2+1+1	270	NLO $SU(2)$ chiral fit.
ETM 10	[100]	2+1+1	270	$SU(2)$ NLO and NNLO fits.

Table 93: Chiral extrapolation/minimum pion mass in $N_f = 2+1+1$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
JLQCD 16B	[37]	2+1	225.8(0.3)	NLO $SU(2)$ ChPT
RBC/UKQCD 15E	[103]	2+1	117.3(4.4)	GMOR for Σ, NNLO PQ $SU(2)$ χPT.
JLQCD 15A	[102]	2+1	290	Dynamical overlap, NNLO $SU(3)$.
RBC/UKQCD 14B	[5]	2+1	139.2	GMOR for Σ, global cont./chiral fit.
JLQCD 14	[111]	2+1	99	ϵ expansion.
Boyle 14	[104]	2+1	171	Combines latt/pheno.
BMW 13	[105]	2+1	120	NLO and NNLO $SU(2)$ fits tested with x and ξ expansion.
RBC/UKQCD 12	[6]	2+1	293 plus run at 171, 246	NLO $SU(2)$ χPT incl. finite-V and some discr. effects
Borsanyi 12	[106]	2+1	135	NNLO $SU(2)$ chiral fit.
NPLQCD 11	[107]	2+1	235	NNLO $SU(2)$ mixed action χPT.
PACS-CS 11A	[114]	2+1	296	Additional test runs at physical point.
JLQCD/TWQCD 09, 10A	[110]	2+1,3	100(ϵ-reg.), 290(p-reg.)	$N_f = 2+1$ runs both in ϵ- and p-regime; $N_f = 3$ runs only in p-regime. NLO χPT fit of the spectral density interpolating the two regimes.
RBC/UKQCD 09, 10A	[14, 112]	2+1	290–420	Valence pions mass is 225-420 MeV. NLO $SU(2)$ χPT fit.
MILC 09, 09A, 10, 10A	[9, 19, 20, 84]	2+1	258	Lightest Nambu-Goldstone mass is 224 MeV and lightest RMS mass is 258 MeV (at 0.06 fm).
TWQCD 08	[113]	2+1	$m_{ud} = m_s/4$, $m_s \sim$ phys.	Quark condensate extracted from topological susceptibility, LO chiral fit.
PACS-CS 08	[21]	2+1	156	Simulation at physical point.
RBC/UKQCD 08	[22]	2+1	330	Lightest velence pion mass is 242 MeV.
RBC/UKQCD 08A	[115]	2+1	330	Computed at one pion mass.
NPLQCD 06	[89]	2+1	460	Value refers to lightest RMS mass at $a = 0.125$ fm as quoted in [19].
LHP 04	[116]	2+1	318	Vector meson dominance fit.

Table 94: Chiral extrapolation/minimum pion mass in 2+1 determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ETMC 15A	[117]	2	134	Simulation at physical point.
Gülpers 15	[118]	2	193	NLO $SU(2)$ fit.
Engel 14	[119]	2	193	NLO $SU(2)$ fit, Dirac op. and GMOR for Σ.
Gülpers 13	[120]	2	280	NLO χPT fit.
Brandt 13	[121]	2	280	Configurations are shared with CLS.
QCDSF 13	[122]	2	130	Fit with χPT + analytic.
ETM 12, 13	[99, 129]	2	260	Confs shared with ETM 09C.
Bernardoni 11	[123]	2	312	Overlap varence + $\mathcal{O}(a)$ improved Wilson sea, mixed regime χPT.
TWQCD 11	[125]	2	230	NLO $SU(2)$ χPT fit.
TWQCD 11A	[124]	2	220	NLO χPT.
Bernardoni 10	[126]	2	297, 377, 426	NLO $SU(2)$ fit of χ_{top}.
JLQCD/TWQCD 10A	[110]	2	$\sqrt{2m_{\text{min}}\Sigma}/F$=120 ($\epsilon$-reg.), 290 ($p$-reg.)	Data both in the p and ϵ-regime. NLO chiral fit of the spectral density interpolating the two regimes.
JLQCD/TWQCD 09	[133]	2	290	LECs extracted from NNLO chiral fit of vector and scalar radii $\langle r^2\rangle^\pi_{V,S}$.
ETM 09B	[127]	2	$\sqrt{2m_{\text{min}}\Sigma}/F$=85	NLO $SU(2)$ ϵ-regime fit.
ETM 09C	[128]	2	280	NNLO $SU(2)$ fit.
ETM 08	[130]	2	260	From pion form factor using NNLO χPT and exp. value of $\langle r^2\rangle^\pi_S$.
JLQCD/TWQCD 08A JLQCD 08A	[33] [134]	2	290	NNLO $SU(2)$ fit.
CERN 08	[135]	2	$m_{q,\text{min}}$=13 MeV	NLO $SU(2)$ fit for the mode number.
Hasenfratz 08	[136]	2	$\sqrt{2m_{\text{min}}\Sigma}/F$=220	NLO $SU(2)$ ϵ-regime fit.
JLQCD/TWQCD 07	[132]	2	$\sqrt{2m_{\text{min}}\Sigma}/F$=120	NLO $SU(2)$ ϵ-regime fit.
JLQCD/TWQCD 07A	[131]	2	$m_{ud} = m_s/6 - m_s$	Σ from χ_t, LO chiral fit.
CERN-TOV 06	[138]	2	403, 381, 377	NLO $SU(2)$ fit.
QCDSF/UKQCD 06A	[139]	2	400	Several fit functions to extrapolate the pion form factor.

Table 95: Chiral extrapolation/minimum pion mass in $N_f = 2$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
HPQCD 15B	[98]	2+1+1		4.8	
HPQCD 13A	[75]	2+1+1	4.8–5.5	3.3	3 volumes are compared.
ETM 13	[99]	2+1+1	1.9–2.8	3.0	4 volumes compared.
ETM 10, 11	[79, 100]	2+1+1	1.9-2.8	3.0	FSE estimate using [43]. $M_{\pi^+}L \gtrsim 4$, but $M_{\pi^0}L \sim 2$.
JLQCD 16B	[37]	2+1		4.1743	2 volumes.
RBC/UKQCD 15E	[103]	2+1		3.78	1 volume.
JLQCD 15A	[102]	2+1		3.88	1 volume.
RBC/UKQCD 14B	[5]	2+1		5.476	1 volume.
JLQCD 14	[111]	2+1		1.8	ϵ-regime
Boyle 14	[104]	2+1		4.6	1 volume.
BMW 13	[105]	2+1	2.1	3.0	3 volumes are compared.
RBC/UKQCD 12	[6]	2+1	2.7–4.6	> 4	FSE seem to be very small.
Borsanyi 12	[106]	2+1	3.9	3.3	Expected to be less than 1%.
NPLQCD 11	[107]	2+1	2.5–3.5	3.6	Expected to be less than 1%.
MILC 09, 09A, 10, 10A	[9, 19, 20, 84]	2+1	2.52	3.5–4.11	$L \geq 2.9$ fm for lighter masses.
JLQCD/TWQCD 09, 10A	[110]	2+1, 3	1.9, 2.7		2 volumes are compared for a fixed quark mass.
RBC/UKQCD 09, 10A	[14, 112]	2+1	2.7	$\simeq 4$	FSE estimated using χPT.
TWQCD 08	[113]	2+1	1.95	-	No estimate of FSE.
PACS-CS 08, 11A	[21, 114]	2+1	2.9	2.3	FSE is the main concern of the authors. Additional test runs on 64^4.
RBC/UKQCD 08	[22]	2+1	2.74	4.6	FSE by means of χPT.
RBC/UKQCD 08A	[115]	2+1	2.74	4.6	FSE estimated to be $< 1\%$.
NPLQCD 06	[89]	2+1	2.5	3.7	Value refers to lightest valence pion mass.
LHP 04	[116]	2+1	$\simeq 2.4$	3.97	Value refers to domain-wall valence pion mass.

Table 96: Finite volume effects in $N_f = 2+1+1$ and $2+1$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETMC 15A	[117]	2		4.39	2 volumes.
Gülpers 15	[118]	2		4.09	3 volumes, CLS confs.
Engel 14	[119]	2		4.2	3 volumes, CLS confs.
Gülpers 13	[120]	2	4–6	4.3	Configs. shared with CLS.
Brandt 13	[121]	2	~ 5	4	Configs. shared with CLS.
QCDSF 13	[122]	2	1.8–2.4	2.7	NLO χPT is used for FSE.
Bernardoni 11	[123]	2	1.56	2.5	Mixed regime χPT for FSE used.
TWQCD 11	[125]	2	1.65	1.92	$SU(2)$ χPT is used for FSE.
TWQCD 11A	[124]	2	1.65	1.8	No estimate of FSE.
Bernardoni 10	[126]	2	1.88	2.8	FSE included in the NLO chiral fit.
JLQCD/TWQCD 10A	[110]	2	1.8-1.9		FSE estimated from different topological sectors.
JLQCD/TWQCD 09	[133]	2	1.89	2.9	FSE by NLO χPT, Additional FSE for fixing topology [140].
ETM 09B	[127]	2	1.3, 1.5	ϵ-regime	Topology: not fixed. 2 volumes.
ETM 09C, 12, 13	[99, 128, 129]	2	2.0-2.5	3.2–4.4	Several volumes. Finite-volume effects estimated through [43].
ETM 08	[130]	2	2.1, 2.8	3.4, 3.7	Only data with $M_\pi L \gtrsim 4$ are considered.
JLQCD/TWQCD 08A JLQCD 08A	[33] [134]	2	1.89	2.9	FSE estimates through [43]. Additional FSE for fixing topology [140].
CERN 08	[135]	2	1.88, 2.51	-	Two volumes compared.
Hasenfratz 08	[136]	2	1.84, 2.77	ϵ-regime	Topology: not fixed, 2 volumes.
JLQCD/TWQCD 07	[132]	2	1.78	ϵ-regime	Topology: fixed to $\nu = 0$.
JLQCD/TWQCD 07A	[131]	2	1.92	-	Topology fixed to $\nu = 0$ [140].
CERN-TOV 06	[138]	2	1.72, 1.67, 1.88	3.5, 3.2, 3.6	No estimate for FSE.
QCDSF/UKQCD 06A	[139]	2	1.4-2.0	3.8	NLO χPT estimate for FSE [141].

Table 97: Finite volume effects in $N_f = 2$ determinations of the Low-Energy Constants.

Collab.	Ref.	N_f	Description
HPQCD 15B	[98]	2+1+1	—
HPQCD 13A	[75]	2+1+1	—
ETM 10,11, 13	[79, 99, 100]	2+1+1	Non-perturbative
JLQCD 16B	[37]	2+1	Non-perturbative
RBC/UKQCD 15E	[103]	2+1	RI-SMOM
JLQCD 15A	[102]	2+1	RI-MOM
RBC/UKQCD 14B	[5]	2+1	RI-SMOM
JLQCD 14	[111]	2+1	—
Boyle 14	[104]	2+1	—
BMW 13	[105]	2+1	Non-perturbative
RBC/UKQCD 12	[6]	2+1	Non-perturbative (RI/SMOM)
Borsanyi 12	[106]	2+1	Indirectly non-perturbative through [12] for Σ; no renormalization needed for F, since only F_π/F computed and scale set through F_π.
NPLQCD 11	[107]	2+1	Not needed (no result for Σ).
JLQCD/TWQCD 10A	[110]	2+1, 3	Non-perturbative
MILC 09, 09A, 10, 10A	[9, 19, 20, 84]	2+1	2 loop
RBC/UKQCD 10A	[14]	2+1	Non-perturbative
JLQCD 09	[109]	2+1	Non-perturbative
TWQCD 08	[113]	2+1	Non-perturbative
PACS-CS 08	[21]	2+1	1 loop
RBC/UKQCD 08, 08A	[22, 115]	2+1	Non-perturbative
NPLQCD 06	[89]	2+1	—
LHP 04	[116]	2+1	—
All collaborations		2	Non-perturbative

Table 98: Renormalization in determinations of the Low-Energy Constants.

B.4 Notes to section 6 on Kaon mixing

B.4.1 Kaon B-parameter B_K

Collab.	Ref.	N_f	a [fm]	Description
ETM 15	[142]	2+1+1	0.09, 0.08, 0.06	Combined chiral and continuum extrapolation. Systematic error of 2.0% is obtained from the distribution of results over analyses which differ by $O(a^2)$ effects.
RBC/UKQCD 16	[143]	2+1	0.111, 0.083	Systematic uncertainty of 1.3% obtained from half the difference between the results on the fine lattice spacing and the continuum limit.
SWME 15A	[144]	2+1	0.12, 0.09, 0.06, 0.045	The three finest lattice spacings are used for the combined chiral and continnum extrapolation. Residual combined discretization, sea-quark extrapolation and α_s matching error of 4.4% from difference between linear fit in a^2, $m_{\rm sea}$ and a fit where α_s dependence is added.
RBC/UKQCD 14B	[5]	2+1	0.111, 0.083, 0.063, 0.114, 0.084	The three first lattice spacings use different action from the last two ones. Combined continuum and chiral fits.
SWME 14	[145]	2+1	0.082, 0.059, 0.044	Residual combined discretization and sea-quark extrapolating error of 0.9% from difference between linear fit in a^2, $m_{\rm sea}$ and a constrained nine-parameter extrapolation.
SWME 13A	[146]	2+1	0.09, 0.06, 0.045	Residual combined discretization, sea-quark extrapolation and α_s matching error of 4.4% from difference between linear fit in a^2, $m_{\rm sea}$ and a fit where α_s dependence is added.
SWME 13	[147]	2+1	0.12, 0.09, 0.06, 0.045	Continuum extrapolation with the coarsest lattice spacing omitted; residual combined discretization and sea-quark extrapolationg error of 1.1% from difference between linear fit in a^2, $m_{\rm sea}$ and a constrained nine-parameter extrapolation.
RBC/UKQCD 12A	[6]	2+1	0.146, 0.114, 0.087	Coarsest lattice spacing uses different action. Combined continuum and chiral fits.

Table 99: Continuum extrapolations/estimation of lattice artifacts in determinations of B_K.

Collab.	Ref.	N_f	a [fm]	Description
Laiho 11	[8]	2+1	0.12, 0.09, 0.06	Combined continuum and chiral extrapolation based on SU(3) mixed-action partially quenched χPT.
SWME 11, 11A	[148, 149]	2+1	0.12, 0.09, 0.06, 0.045	Continuum extrapolation with the coarsest lattice spacing omitted; residual discretization error of 1.9% from difference between fit to a constant and a constrained five-parameter extrapolation.
BMW 11	[150]	2+1	0.093, 0.077, 0.065, 0.054	Combined continuum and chiral extrapolation; discretization error of 0.1% from comparison of $O(\alpha_s a)$ and $O(a^2)$ extrapolations.
RBC/UKQCD 10B	[151]	2+1	0.114, 0.087	Two lattice spacings. Combined chiral and continuum fits.
SWME 10	[152]	2+1	0.12, 0.09, 0.06	Continuum extrapolation of results obtained at four lattice spacings; residual discretization error of 0.21% from difference to result at smallest lattice spacing.
Aubin 09	[153]	2+1	0.12, 0.09	Two lattice spacings; quote 0.3% discretization error, estimated from various a^2-terms in fit function
RBC/UKQCD 07A, 08	[22, 154]	2+1	0.114(2)	Single lattice spacing; quote 4% discretization error, estimated from the difference between computed and experimental values of f_π.
HPQCD/UKQCD 06	[155]	2+1	0.12	Single lattice spacing; 3% discretization error quoted without providing details.
ETM 12D	[156]	2	0.1, 0.09, 0.07, 0.05	Four lattice spacings; systematic quoted obtained from the difference between the finest lattice spacing and the continuum limit and comparing results using two evaluations of the RCs that differ by $O(a^2)$ effects.
ETM 10A	[157]	2	0.1, 0.09, 0.07	Three lattice spacings; 1.2% error quoted.
JLQCD 08	[158]	2	0.118(1)	Single lattice spacing; no error quoted.
RBC 04	[159]	2	0.117(4)	Single lattice spacing; no error quoted.
UKQCD 04	[160]	2	0.10	Single lattice spacing; no error quoted.

Table 99: (cntd.) Continuum extrapolations/estimation of lattice artifacts in determinations of B_K.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
ETM 15	[142]	2+1+1	$245, 239, 211$	Each $M_{\pi,\rm min}$ entry corresponds to a different lattice spacing. Simultaneous chiral & continuum extrapolations, based on polynomial and χPT at NLO, are carried out leads to systematic error of 0.8% .
RBC/UKQCD 16	[143]	2+1	337, 302	Chiral extrapolations based on polynomial and SU(2)-χPT fits at NLO. A systematic uncertainty of 0.4% is quoted, which is half the difference between the two results.
SWME 15A	[144]	2+1	222/372, 206/174, 195/222, 206/316	Valence/sea RMS $M_{\pi,\rm min}$ entries correspond to the four lattice spacings (the last three are used for the chiral-continuum extrapolation). Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.05% from changing the Bayesian priors and fit method.
RBC/UKQCD 14B	[5]	2+1	$337, 302, 371, 139, 139$	$M_{\pi,\rm min}$ entries correspond to the five lattice spacings. Combined chiral & continuum extrapolation, using $M_\pi < 260$ MeV and $M_\pi < 370$ MeV.
SWME 14	[145]	2+1	206/174, 195/222, 207/316	Valence/sea RMS $M_{\pi,\rm min}$ entries correspond to the three lattice spacings. Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.1% error from doubling the widths of Bayesian priors.
SWME 13A	[146]	2+1	207/243, 196/262, 207/316	Valence/sea RMS $M_{\pi,\rm min}$ entries correspond to the three lattice spacings. Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.1% from doubling the widths of Bayesian priors.
SWME 13	[147]	2+1	442/445, 299/273, 237/256, 222/334	Valence/sea RMS $M_{\pi,\rm min}$ entries correspond to the four lattice spacings. Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.33% error from doubling the widths of Bayesian priors.

Table 100: Chiral extrapolation/minimum pion mass in determinations of B_K.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
RBC/UKQCD 12A	[6]	2+1	140/170, 240/330, 220/290	Valence/sea $M_{\pi,\rm min}$ entries correspond to the three lattice spacings. Combined chiral & continuum extrapolation, using $M_\pi <$ 350 MeV.
Laiho 11	[8]	2+1	210/280	$M_{\pi,\rm min}$ entries correspond to the smallest valence/sea quark masses. Chiral & continuum fits based on NLO mixed action χPT, including a subset of NNLO terms. Systematic error estimated from spread arising from variations in the fit function.
SWME 11, 11A	[148, 149]	2+1	442/445, 299/325, 237/340, 222/334	Valence/sea RMS $M_{\pi,\rm min}$ entries correspond to the four lattice spacings. Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.33% error from doubling the widths of Bayesian priors.
BMW 11	[150]	2+1	219, 182, 120, 131	$M_{\pi,\rm min}$ entries correspond to the four lattice spacings used in the final result. Combined fit to the chiral and continuum behaviour. Systematics investigated by applying cuts to the maximum pion mass used in fits. Uncertainty of 0.1% assigned to chiral fit.
RBC/UKQCD 10B	[151]	2+1	240/330, 220/290	Valence/sea $M_{\pi,\rm min}$ entries correspond to the two lattice spacings. Combined chiral and continuum extrapolations.
SWME 10	[152]	2+1	442/445, 299/325, 237/340	Valence/sea $M_{\pi,\rm min}$ entries correspond to the three lattice spacings. Chiral extrapolations based on SU(2) staggered χPT at NLO, including some analytic NNLO terms. SU(3) staggered χPT as cross-check. Combined 1.1% error from various different variations in the fit procedure.
Aubin 09	[153]	2+1	240/370	$M_{\pi,\rm min}$ entries correspond to the smallest valence/sea quark masses. Chiral & continuum fits based on NLO mixed action χPT at NLO, including a subset of NNLO terms. Systematic error estimated from spread arising from variations in the fit function.

Table 100: (cntd.) Chiral extrapolation/minimum pion mass in determinations of B_K.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
RBC/UKQCD 07A, 08	[22, 154]	2+1	330	Fits based on SU(2) PQχPT at NLO. Effect of neglecting higher orders estimated at 6% via difference between fits based on LO and NLO expressions.
HPQCD/UKQCD 06	[155]	2+1	360	3% uncertainty from chiral extrapolation quoted, without giving further details.
ETM 12D	[156]	2	400, 280, 300, 280	Each $M_{\pi,\text{min}}$ entry corresponds to a different lattice spacing. Simultaneous chiral & continuum extrapolations, based on polynomial and χPT at NLO, are carried out. Systematic error from several sources, including lattice calibration, quark mass calibration, chiral and continuum extrapolation etc., estimated at 3.0%.
ETM 10A	[157]	2	400, 280, 300	Each $M_{\pi,\text{min}}$ entry corresponds to a different lattice spacing. Simultaneous chiral & continuum extrapolations, based on χPT at NLO, are carried out. Systematic error from several sources, including lattice calibration, quark mass calibration, chiral and continuum extrapolation etc., estimated at 3.1%.
JLQCD 08	[158]	2	290	Fits based on NLO PQχPT. Range of validity investigated. Fit error included in statistical uncertainty.
RBC 04	[159]	2	490	Fits based on NLO PQχPT. Fit error included in statistical uncertainty.
UKQCD 04	[160]	2	780	Fits to continuum chiral behaviour at fixed sea quark mass. Separate extrapolation in sea quark mass. Fit error included in overall uncertainty.

Table 100: (cntd.) Chiral extrapolation/minimum pion mass in determinations of B_K.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 15	[142]	2+1+1	2.1–2.8, 2.6, 3.0	3.5, 3.2, 3.2	Each L entry corresponds to a different lattice spacing, with two volumes at the coarsest lattice spacing; results from these two volumes at $M_\pi \sim 280$ MeV are compatible.
RBC/UKQCD 16	[143]	2+1	2.7, 2.7	4.5, 4.0	Finite volume effects are found to be negligible compared to the systematic errors and are thus omitted in the final error budget.
SWME 15A	[144]	2+1	2.4–3.4, 2.5–5.8, 2.9–3.9, 2.9	$\gtrsim 3.8$	L entries correspond to the four lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO SU(2) SχPT.
RBC/UKQCD 14B	[5]	2+1	2.7, 2.7, 2.0, 5.5, 5.3	$\gtrsim 3.8$	L entries correspond to the five lattice spacings. Finite volume effects estimated using NLO χPT; negligible with comparison to the statistical error.
SWME 14	[145]	2+1	2.8–5.4, 2.8–3.8, 2.8	5.6, 3.7, 2.9	L entries correspond to the three lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO χPT.
SWME 13A	[146]	2+1	2.4–3.4, 2.8–3.8, 2.8	3.5, 3.3, 2.9	L entries correspond to the three lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO χPT.
SWME 13	[147]	2+1	2.4–3.3, 2.4–5.5, 2.8–3.8, 2.8	$\gtrsim 3.2$	L entries correspond to the four lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO χPT.
RBC/UKQCD 12A	[6]	2+1	4.6, 2.7, 2.8	$\gtrsim 3.2$	L entries correspond to the three lattice spacings. Finite volume effects estimated using NLO χPT.
Laiho 11	[8]	2+1	2.4, 3.4, 3.8	$\gtrsim 3.5$	L entries correspond to the three lattice spacings. Finite volume effects estimated using NLO χPT.
SWME 11, 11A	[148, 149]	2+1	2.4/3.3, 2.4, 2.8, 2.8	$\gtrsim 3.2$	L entries correspond to the four lattice spacings, with two volumes at the coarsest lattice. Finite-volume effects estimated using NLO χPT.

Table 101: Finite volume effects in determinations of B_K. If partially-quenched fits are used, the quoted $M_{\pi,\min}L$ is for lightest valence (RMS) pion.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
BMW 11	[150]	2+1	6.0, 4.9, 4.2, 3.5	$\gtrsim 3.8, 3.0$	L entries correspond to the four lattice spacings, and are the largest of several volumes at each a. $M_{\pi,\min}L \approx 3.0$ for the ensemble at $a \approx 0.08$ fm. Finite volume effects estimated in χPT and by combined fit to multiple volumes.
RBC/UKQCD 10B	[151]	2+1	2.7, 2.8	$\gtrsim 3.1$	L entries correspond to the three lattice spacings. Finite volume effects estimated using NLO χPT.
SWME 10	[152]	2+1	2.4/3.3, 2.4,2.8	$\gtrsim 3.4$	L entries correspond to the three lattice spacings, with two volumes for the coarsest spacing. Finite-volume error of 0.9% estimated from difference obtained these two volumes.
Aubin 09	[153]	2+1	2.4, 3.4	3.5	L entries correspond to the two lattice spacings. Keep $m_\pi L \gtrsim 3.5$; no comparison of results from different volumes; 0.6% error estimated from mixed action χPT correction.
RBC/UKQCD 07A, 08	[22, 154]	2+1	1.83/2.74	4.60	Each L entry corresponds to a different volume at the same lattice spacing; 1% error from difference in results on two volumes.
HPQCD/UKQCD 06	[155]	2+1	2.46	4.49	Single volume; no error quoted.
ETM 12D	[156]	2	2.1, 2.2/2.9, 2.2, 2.6	5, 3.3/4.3, 3.3, 3.5	Each L entry corresponds to a different lattice spacing, with two volumes at the second less coarse lattice spacing. Results from these two volumes at $M_\pi \sim 300$ MeV are compatible.
ETM 10A	[157]	2	2.1, 2.2/2.9, 2.2	5, 3.3/4.3, 3.3	Each L entry corresponds to a different lattice spacing, with two volumes at the intermediate lattice spacing. Results from these two volumes at $M_\pi \sim 300$ MeV are compatible.
JLQCD 08	[158]	2	1.89	2.75	Single volume; data points with $m_{\rm val} < m_{\rm sea}$ excluded; 5% error quoted as upper bound of PQχPT estimate of the effect.
RBC 04	[159]	2	1.87	4.64	Single volume; no error quoted.
UKQCD 04	[160]	2	1.6	6.51	Single volume; no error quoted.

Table 101: (cntd.) Finite volume effects in determinations of B_K

Collab.	Ref.	N_f	Ren.	running match.	Description
ETM 15	[142]	2+1+1	RI	PT1ℓ	Uncertainty from RI renormalization estimated at 2%. Additional error of 0.6% for the conversion to $\overline{\rm MS}$.
RBC/UKQCD 16	[143]	2+1+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate 2% systematic error in conversion to $\overline{\rm MS}$.
SWME 15A	[144]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
RBC/UKQCD 14B	[5]	2+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate 2% systematic error in conversion to $\overline{\rm MS}$.
SWME 14	[145]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
SWME 13A	[146]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% (in combination with systematic uncertainty from CL and chiral extrapolation fit) by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
SWME 13	[147]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
RBC/UKQCD 12A	[6]	2+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate 2% systematic error in conversion to $\overline{\rm MS}$.
Laiho 11	[8]	2+1	RI	PT1ℓ	Total uncertainty in matching & running of 3%. Perturbative truncation error in the conversion to $\overline{\rm MS}$, RGI schemes is dominant uncertainty.

Table 102: Running and matching in determinations of B_K for $N_f = 2+1+1$ and $N_f = 2+1$.

Collab.	Ref.	N_f	Ren.	running match.	Description
SWME 11, 11A	[148, 149]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
BMW 11	[150]	2+1	RI	PT1ℓ	Uncertainty of 0.05% in the determination of the renormalization factor included. 1% error estimated due to truncation of perturbative matching to $\overline{\text{MS}}$ and RGI schemes at NLO.
RBC/UKQCD 10B	[151]	2+1	RI	PT1ℓ	Variety of different RI-MOM schemes including non-exceptional momenta. Residual uncertainty of 2% uncertainty in running & matching.
SWME 10	[152]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 5.5% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
Aubin 09	[153]	2+1	RI	PT1ℓ	Total uncertainty in matching & running of 3.3%, estimated from a number of sources, including chiral extrapolation fit ansatz for n.p. determination, strange sea quark mass dependence, residual chiral symmetry breaking, perturbative matching & running.
RBC/UKQCD 07A, 08	[22, 154]	2+1	RI	PT1ℓ	Uncertainty from n.p. determination of ren. factor included in statistical error; 2% systematic error from perturbative matching to $\overline{\text{MS}}$ estimated via size of correction itself.
HPQCD/UKQCD 06	[155]	2+1	PT1ℓ	PT1ℓ	Uncertainty due to neglecting 2-loop order in perturbative matching and running estimated by multiplying result by α^2.

Table 102: (cntd.) Running and matching in determinations of B_K for $N_f = 2+1+1$ and $N_f = 2+1$.

Collab.	Ref.	N_f	Ren.	running match.	Description
ETM 12D	[156]	2	RI	PT1ℓ	Uncertainty from RI renormalization estimated at 2.5%.
ETM 10A	[157]	2	RI	PT1ℓ	Uncertainty from RI renormalization estimated at 2.5%.
JLQCD 08	[158]	2	RI	PT1ℓ	Uncertainty from n.p. determination of ren. factor included in statistical error; 2.3% systematic error from perturbative matching to $\overline{\mathrm{MS}}$ estimated via size of correction itself.
RBC 04	[159]	2	RI	PT1ℓ	Uncertainty from n.p. determination of ren. factor included.
UKQCD 04	[160]	2	PT1ℓ	PT1ℓ	No error quoted.

Table 103: Running and matching in determinations of B_K for $N_f = 2$.

B.4.2 Kaon BSM B-parameters

Collab.	Ref.	N_f	a [fm]	Description
ETM 15	[142]	2+1+1	0.09, 0.08, 0.06	Combined chiral and continuum extrapolation. Systematic errors to B_i from about 4% to 6% are obtained from the distribution of results over analyses which differ by $O(a^2)$ effects.
RBC/UKQCD 16	[143]	2+1	0.111, 0.083	Systematic uncertainty of 1.3% obtained from half the difference between the results on the fine lattice spacing and the continuum limit.
SWME 15A	[144]	2+1	0.12, 0.09, 0.06, 0.045	The three finest lattice spacings are used for the combined chiral and continnum extrapolation. Residual combined discretization, sea-quark extrapolation and α_s matching error from about 4.4% to 9.6% is reported for B_i and is obtained from the difference between linear fit in a^2, $m_{\rm sea}$ and a fit where α_s dependence is added.
SWME 14C	[161]	2+1	0.082, 0.059, 0.044	Residual combined discretization and sea-quark extrapolationg error of 1–8% from difference between linear fit in a^2, $m_{\rm sea}$ and a constrained nighteen-parameter extrapolation.
SWME 13A	[146]	2+1	0.09, 0.06, 0.045	Residual combined discretization, sea-quark extrapolation and α_s matching error for B_i varies from 4.5% to -5.7% , from difference between linear fit in a^2, $m_{\rm sea}$ and a fit where α_s dependence is added.
RBC/UKQCD 12E	[162]	2+1	0.087	Computation at only one value of the lattice spacing. Estimate for the systematic discretisation error of about 1.5% based on the corresponding estimate from the B_K computation.
ETM 12D	[156]	2	0.1, 0.09, 0.07, 0.05	Four lattice spacings; Estimates of systematic uncertainties obtained from the half difference of the distance between the finest lattice spacing and the continuum limit.

Table 104: Continuum extrapolations/estimation of lattice artifacts in determinations of the BSM B_i parameters.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
ETM 15	[142]	2+1+1	245, 239, 211	Each $M_{\pi,\mathrm{min}}$ entry corresponds to a different lattice spacing. Simultaneous chiral & continuum extrapolations, based on polynomial and χPT at NLO, are carried out leads to systematic errors of $1.1-2.6\%$ depending on the bag parameter.
RBC/UKQCD 16	[143]	2+1	337, 302	Chiral extrapolations based on polynomial and SU(2)-χPT fits at NLO. A systematic uncertainty of 0.4% is quoted, which is half the difference between the two results.
SWME 15A	[144]	2+1	222/372, 206/174, 195/222, 206/316	Valence/sea RMS $M_{\pi,\mathrm{min}}$ entries correspond to the four lattice spacings (the last three are used for the chiral-continuum extrapolation). Chiral extrapolations based on SU(2) staggered χPT at NNLO (with some coefficients fixed by Bayesian priors), and also including one analytic NNNLO term. Residual error of 0.4-1.2% depending on the bag parameter from changing the Bayesian priors and fit method.
SWME 14C	[161]	2+1	206/174, 195/222, 207/316	Valence/sea RMS $M_{\pi,\mathrm{min}}$ entries correspond to the three lattice spacings. Chiral extrapolations performed via B_i-ratios that do not show SU(2) NLO χPT contribution and assuming various terms up to NNLO (with some coefficients fixed by Bayesian priors).
SWME 13A	[146]	2+1	207/243, 196/262, 207/316	Valence/sea RMS $M_{\pi,\mathrm{min}}$ entries correspond to the three lattice spacings. Chiral extrapolations performed via B_i-ratios that do not show SU(2) NLO χPT contribution and assuming various terms up to NNLO (with some coefficients fixed by Bayesian priors). Residual error in the valence of about 0.1% from doubling the widths of Bayesian priors. In the sea a combined error with the matching procedure of 4.4-5.6% is reported.
RBC/UKQCD 12E	[162]	2+1	290/290	Chiral extrapolations based on polynomial and χPT fits at NLO are carried out. Central values are obtained from polynomial fits. Mild dependence on the quark mass. Systematic uncertainties are estimated to about 4% for all B_i's.
ETM 12D	[156]	2	400, 270, 300, 270	Each $M_{\pi,\mathrm{min}}$ entry corresponds to a different lattice spacing. Simultaneous chiral & continuum extrapolations, based on polynomial and χPT at NLO, are carried out.

Table 105: Chiral extrapolation/minimum pion mass in determinations of the BSM B_i parameters.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
ETM 15	[142]	2+1+1	2.1–2.8, 2.6, 3.0	$3.5, 3.2, 3.2$	Each L entry corresponds to a different lattice spacing, with two volumes at the coarsest lattice spacing; results from these two volumes at $M_\pi \sim 280$ MeV are compatible.
RBC/UKQCD 16	[143]	2+1	2.7, 2.7	4.5, 4.0	Finite volume effects are found to be negligible compared to systematic errors and are thus omitted in the final error budget.
SWME 15A	[144]	2+1	2.4–3.4, 2.5–5.8, 2.9–3.9, 2.9	$\gtrsim 3.8$	L entries correspond to the four lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO SU(2) SχPT.
SWME 14C	[161]	2+1	2.8–5.4, 2.8–3.8, 2.8	$5.6, 3.7, 2.9$	L entries correspond to the three lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO χPT.
SWME 13A	[146]	2+1	2.4–3.4, 2.8–3.3, 2.8	$3.5, 3.3, 2.9$	L entries correspond to the three lattice spacings, with several volumes in most cases. Finite-volume effects estimated using NLO χPT.
RBC/UKQCD 12E	[162]	2+1	2.8	$\gtrsim 4.0$	The L value corresponds to the unique lattice spacing. Finite volume effects, estimated using NLO χPT are small, as it has also been found in the B_K computation, and they have thus been neglected in the final error budget analysis.
ETM 12D	[156]	2	2.1, 2.2/2.9, 2.2, 2.6	5, 3.3/4.3, 3.3, 3.5	Each L entry corresponds to a different lattice spacing, with two volumes at the second less coarse lattice spacing. Results from these two volumes at $M_\pi \sim 300$ MeV are compatible.

Table 106: Finite volume effects in determinations of the BSM B_i parameters. If partially-quenched fits are used, the quoted $M_{\pi,\rm min}L$ is for lightest valence (RMS) pion.

Collab.	Ref.	N_f	Ren.	running match.	Description
ETM 15	[142]	2+1+1	RI	PT1ℓ	Uncertainty from RI renormalization combined with discretisation effects estimates are reported to be from about 4% to 6% . Additional error from 1.8 to 3.9% (dependind on the bag parameter) for the conversion to $\overline{\text{MS}}$ at the scale of 3 GeV.
RBC/UKQCD 16	[143]	2+1+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate systematic error in conversion to $\overline{\text{MS}}$, which varies from 1–4%, depending on the four-quark operator.
SWME 15A	[144]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated from about 4.4% to 9.6% (depending on the bag parameter) by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
SWME 14C	[161]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
SWME 13A	[146]	2+1	PT1ℓ	PT1ℓ	Uncertainty from neglecting higher orders estimated at 4.4% (in combination with systematic uncertainty from CL and chiral extrapolation fit) by identifying the unknown 2-loop coefficient with result at the smallest lattice spacing.
RBC/UKQCD 12E	[162]	2+1	RI	PT1ℓ	Computation in RI-MOM scheme. Systematic error from the conversion to $\overline{\text{MS}}$ is estimated by taking the half of the difference between the LO and the NLO result.
ETM 12D	[156]	2	RI	PT1ℓ	Uncertainty from RI renormalization estimated at 2.5%.

Table 107: Running and matching in determinations of the BSM B_i parameters.

B.5 Notes to section 7 on D-meson decay constants and form factors

In the following, we summarize the characteristics (lattice actions, pion masses, lattice spacings, etc.) of the recent $N_f = 2 + 1 + 1$, $N_f = 2 + 1$ and $N_f = 2$ runs. We also provide brief descriptions of how systematic errors are estimated by the various authors. We focus on calculations with either preliminary or published quantitative results.

B.5.1 $D_{(s)}$-meson decay constants

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
FNAL/MILC 14A	[2]	2+1+1	311, 241, 173, 143	The lightest pions (not RMS) are around 130 MeV. Analyses are performed either by interpolating to the physical point or by using HMrASχPT formulae to include heavier masses and non-unitary points. The latter procedure gives more accurate, and final, results.
ETM 13F ETM 14E	[74, 77]	2+1+1	245, 239, 211	$f_{D_s}\sqrt{m_{D_s}}$ in ETM 13F and f_{D_s}/m_{D_s} in ETM 14E are extrapolated using both a quadratic and a linear fit in m_l plus $\mathcal{O}(a^2)$ terms. Then the double ratio $(f_{D_s}/f_D)/(f_K/f_\pi)$ is fitted in continuum HMχPT, as no lattice spacing dependence is visible within statistical errors.
FNAL/MILC 12B FNAL/MILC 13	[163, 164]	2+1+1	310, 245, 179, 145	Chiral and continuum extrapolations are peformed simultaneously. Central values are produced using a fit function quadratic in a^2 and linear in the sea-quark mass. In FNAL/MILC 13 terms of $\mathcal{O}(a^4)$ are included.

Table 108: Chiral extrapolation/minimum pion mass in $N_f = 2+1+1$ determinations of the D and D_s meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
χQCD 14	[48]	2+1	334, 296	Chiral and continuum extrapolations are peformed simultaneously using linear fits in m_l (and quadratic or including partially quenched chiral logs, in order to assess the systematic error) plus terms up to $\mathcal{O}(a^2)$ and $\mathcal{O}(a^4 m_c^4)$.
HPQCD 12A	[165]	2+1	460, 329	Chiral and continuum extrapolations are peformed simultaneously using PQHMχPT augmented by a dependent terms: $c_0(am_c)^2 + c_1(am_c)^4$.
FNAL/MILC 11	[166]	2+1	570, 440, 320	Chiral and continuum extrapolations are peformed simultaneously using HMχPT for rooted staggered quarks. Effects of hyperfine and flavour splittings are also included.
PACS-CS 11	[167]	2+1	152	Simulations are reweighted in the light- and strange-quark masses to the physical point.
HPQCD 10A	[168]	2+1	542, 460, 329, 258, 334	Chiral and continuum extrapolations are performed simultaneously. Polynomials up to $\left(\frac{m_{q,sea}-m_{q,phys}}{m_{q,phys}}\right)^2$ for $q = s, l$ and up to $(am_c)^8$ are kept.
HPQCD/UKQCD 07	[88]	2+1	542, 460, 329	Combined chiral and continuum extrapolations using HMχPT at NLO augmented by second and third-order polynomial terms in m_q and terms up to a^4.
FNAL/MILC 05	[169]	2+1	$>$ 440 , 440 , 400	Chiral extrapolations are first performed at each lattice spacing uisng NLO HMχPT for rooted staggered quarks. Lattice artefacts are then extrapolated linearly in a^2.

Table 109: Chiral extrapolation/minimum pion mass in $N_f = 2+1$ determinations of the D and D_s meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
TWQCD 14	[170]	2	260	Comparison of NLO HMχPT fits for $f_{D_{(s)}}$ and for $f_{D_{(s)}}\sqrt{m_{D_{(s)}}}$ in order to asses systematic error.
ALPHA 13B	[50]	2	190 , 270	Linear fits (in m_π^2 and in a^2) and partially quenched HMχPT functional forms, including terms linear in a^2, are used in the combined chiral/continuum extrapolation.
ETM 09 ETM 11A ETM 13B	[55, 57, 92]	2	410, 270, 310, 270	$M_{\pi,\text{min}}$ refers to the charged pions. NLO $SU(2)$ HMχPT supplemented by terms linear in a^2 and in $m_D a^2$ is used in the combined chiral/continuum extrapolation. To estimate the systematic due to chiral extrapolation, once $f_{D_s}\sqrt{m_{D_s}}$ and $f_{D_s}\sqrt{m_{D_s}}/(f_D\sqrt{m_D})$ and once $f_{D_s}\sqrt{m_{D_s}}/f_K$ and $f_{D_s}\sqrt{m_{D_s}}/f_K \times f_\pi/(f_D\sqrt{m_D})$ are fitted. In ETM 13 the double ratio $(f_{D_s}/f_D)/(f_K/f_\pi)$ is fitted in HMχPT.

Table 110: Chiral extrapolation/minimum pion mass in $N_f = 2$ determinations of the D and D_s meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
FNAL/MILC 14A	[2]	2+1+1	2.38-4.83, 2.90-5.82, 2.95-5.62, 2.94-5.44	7.6, 7, 4.9, 3.9	3 values of L (2.9, 3.9 and 4.9 fm) at m_π = 220 MeV and a = 0.12 fm.
ETM 13F ETM 14E	[74, 77]	2+1+1	2.13/2.84, 1.96/2.61, 2.97	3.5, 3.2, 3.2	The comparison of two different volumes at the two largest lattice spacings indicates that FV effects are below the statistical errors.
FNAL/MILC 12B FNAL/MILC 13	[163, 164]	2+1+1	2.4/4.8, 2.88/5.76, 2.88/5.76, 2.88/5.76	7.6, 7, 4.9, 3.9	FV errors estimated in χPT at NLO and, in FNAL/MILC 12B, by analyzing otherwise identical ensembles with three different spatial sizes at a = 0.12 fm and $m_l/m_s = 0.1$.

Table 111: Finite volume effects in $N_f = 2+1+1$ determinations of the D and D_s meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the RMS masses are used (where available).

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
χQCD 14	[48]	2+1	2.7, 2.7	4.6, 4.1	No explicit discussion of FV effects.
HPQCD 12A	[165]	2+1	2.4/2.8, 2.4/3.4	6.7, 4.2	FV errors estimated by comparing finite and infinite volume χPT.
FNAL/MILC 11	[166]	2+1	2.4, 2.4/2.88, 2.52/3.6	6.9, 6.4, 5.8	FV errors estimated using finite-volume χPT.
PACS-CS 11	[167]	2+1	2.88	2.2 (before reweighting)	No discussion of FV effects.
HPQCD 10A	[168]	2+1	2.4, 2.4/2.88/3.36, 2.52, 2.88, 2.82	$6.6, 6.7, 4.2, 3.8, 4.8$	FV errors estimated using finite-vs infinite-volume χPT.
HPQCD/UKQCD 07	[88]	2+1	2.4, 2.4/2.88, 2.52	6.6, 6.7, 4.2	FV errors estimated using finite-vs infinite-volume χPT.
FNAL/MILC 05	[169]	2+1	2.8, 2.9, 2.5	$> 6, 6.4, 5$	FV errors estimated to be 1.5% or less from χPT.

Table 112: Finite volume effects in $N_f = 2+1$ determinations of the D and D_s meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the RMS masses are used (where available).

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
TWQCD 14	[170]	2	1.5	1.92	No explicit discussion of FV effects.
ALPHA 13B	[50]	2	2.1/3.1/4.2, 2.3/3.1	4, 4.2	No explicit discussion of FV effects, but $m_\pi L > 4$ always.
ETM 09 ETM 11A ETM 13B	[55, 57, 92]	2	2.4, 2.0/2.7, 2.1, 2.6	5, 3.7, 3.3, 3.5	FV errors are found to be negligible by comparing results at $m_\pi L = 3.3$ and $m_\pi L = 4.3$ for $m_\pi \simeq 310$ MeV.

Table 113: Finite volume effects in $N_f = 2$ determinations of the D and D_s meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the RMS masses are used (where available).

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
FNAL/MILC 14A	[2]	2+1+1	0.15, 0.12, 0.09, 0.06	Interpolations around the physical light masses used to fix the ratio of quark masses. Subsequent chiral and continuum extrapolations for the charm decay constants performed simulatenously using different NLO HMrASχPT fits.	Relative scale through F_{4ps}, the decay constant at valence masses = 0.4 m_s and physical sea-quark masses. Absolute scale set through f_π; the uncertainty is propagated into the final error.
ETM 13F ETM 14E	[74, 77]	2+1+1	0.09, 0.08, 0.06	Chiral and continuum extrapolations performed simultaneously by adding an $\mathcal{O}(a^2)$ term to the chiral fits.	Relative scale set through $M_{c's'}$, the mass of a fictitious meson made of valence quarks of mass $r_0 m_{s'} = 0.22$ and $r_0 m_{c'} = 2.4$. Absoulte scale through f_π.
FNAL/MILC 12B FNAL/MILC 13	[163, 164]	2+1+1	0.15, 0.12, 0.09, 0.06	Chiral and continuum extrapolations performed simultaneously. Central values produced using a fit function quadratic in a^2 and linear in the sea quark mass. In FNAL/MILC 13 terms of $\mathcal{O}(a^4)$ are included.	Absolute scale set through f_π; the uncertainty is propagated into the final error.

Table 114: Lattice spacings and description of actions used in $N_f = 2+1+1$ determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
χQCD 14	[48]	2+1	0.113, 0.085	Chiral and continuum extrapolations performed in global fits including linear terms in m_l and terms up to $\mathcal{O}(a^2)$ and $\mathcal{O}(a^4 m_c^4)$.	Relative scale set through r_0, fixed together with the charm and strange quark masses using m_{D_s}, $m_{D_s^*}$ amd $m_{J/\psi}$ as inputs.
HPQCD 12A	[165]	2+1	0.12, 0.09	Chiral and continuum extrapolations pefomed simultaneously using PQHMχPT augmentd by a dependent terms: $c_0(am_c)^2 + c_1(am_c)^4$.	Relative scale set through r_1; absolute scale from f_π, f_K and the Υ splitting. Uncertainties from both r_1 and r_1/a propagated.
FNAL/MILC 11	[166]	2+1	0.15, 0.12, 0.09	Chiral and continuum extrapolations pefomed simultaneously using one-loop HMχPT for rooted staggered quarks. Effects of hyperfine and flavour splittings are also included.	Relative scale set through r_1 = 0.3117(22). The error in r_1 comes from the spread of different absolute scale determinations using f_π, f_K and the Υ splitting.
PACS-CS 11	[167]	2+1	0.09	Cutoff effects from the heavy-quark action estimated by naive power counting to be at the percent level.	Scale set through m_Ω.
HPQCD 10A	[168]	2+1	0.15, 0.12, 0.09, 0.06, 0.044	Chiral and continuum extrapolations performed simultaneously. Polynomials up to am_c^8 are kept (even powers only).	See the discussion for HPQCD 12A.
HPQCD/UKQCD 07	[88]	2+1	0.15, 0.12, 0.09	Combined chiral and continuum extrapolations using HMχPT at NLO augmented by second and third-order polynomial terms in m_q and terms up to a^4.	Scale set through r_1 obtained from the Υ spectrum using the non-relativistic QCD action for b quarks. Uncertainty propagated among the systematics.
FNAL/MILC 05	[169]	2+1	0.175, 0.121, 0.086	Most light-quark cutoff effects are removed through NLO HMχPT for rooted staggered quarks. Continuum values are then obtained by averaging the $a \approx 0.12$ and $a \approx 0.09$ fm results.	Scale set through r_1 obtained from the Υ spectrum using the non-relativistic QCD action for b quarks.

Table 115: Lattice spacings and description of actions used in $N_f = 2+1$ determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
TWQCD 14	[170]	2	0.061	Uncertainties associated to scale setting and discretization effects estimated by performig the chiral fits once in physical and once in lattice units ($\approx$ 2 MeV on f_{D_s}).	Scale set through the Wilson flow and r_0 set to 0.49 fm.
ALPHA 13B	[50]	2	0.065, 0.048	Linear fits (in m_π^2 and in a^2) and partially quenched HMχPT functional forms, including terms linear in a^2, are used in the combined chiral/continuum extrapolation.	Scale set through f_K.
ETM 09 ETM 11A ETM 13B	[55, 57, 92]	2	0.10, 0.085, 0.065, 0.054	NLO $SU(2)$ HMχPT supplemented by terms linear in a^2 and in $m_D a^2$ is used in the combined chiral/continuum extrapolation.	Scale set through f_π.

Table 116: Lattice spacings and description of actions used in $N_f = 2$ determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 14A	[2]	2+1+1	–	The axial current is absolutely normalized.
ETM 13F, 14E	[74, 77]	2+1+1	–	The axial current is absolutely normalized.
FNAL/MILC 12B, 13	[163, 164]	2+1+1	–	The axial current is absolutely normalized.
χQCD 14	[48]	2+1	RI	The decay constant is extracted from an exact lattice Ward identity and from the NP renormalized axial current.
HPQCD 12A	[165]	2+1	–	The axial current is absolutely normalized.
FNAL/MILC 11	[166]	2+1	mNPR	Two-loop and higher-order perturbative truncation errors estimated to be the full size of the one-loop term.
PACS-CS 11	[167]	2+1	PT1ℓ+NP	Mass dependent part of the renormalization constant of the axial current computed at one-loop; the NP contribution is added in the chiral limit.
HPQCD 10A	[168]	2+1	–	The axial current is absolutely normalized.
HPQCD/UKQCD 07	[88]	2+1	–	The axial current is absolutely normalized.
FNAL/MILC 05	[169]	2+1	mNPR	Errors due to higher order corrections in the perturbative part are estimated to be 1.3%.
TWQCD 14	[170]	2	–	The decay constant is extracted from an exact lattice Ward identity.
ALPHA 13B	[50]	2	SF	NP renormalization and improvement of the axial current (*am* terms included at 1-loop).
ETM 09, 11A, 13B	[55, 57, 92]	2	–	The axial current is absolutely normalized.

Table 117: Operator renormalization in determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 14A	[2]	2+1+1	HISQ (on HISQ)	$0.22 < am_c < 0.84$. Discretization errors estimated to be ≈ 1 MeV using the spread of 108 different chiral/continuum fits (for example by including or not some NNLO discretization effects in HMrASχPT).
ETM 13F, 14E	[74, 77]	2+1+1	tmWil	$0.15 \lesssim am_c \lesssim 0.20$.
FNAL/MILC 12B FNAL/MILC 13	[163, 164]	2+1+1	HISQ (on HISQ)	$0.29 < am_c < 0.7$. Discretization errors estimated using different fit ansätze to be $\approx 1.5\%$ for $f_{D_{(s)}}$.

Table 118: Heavy-quark treatment in $N_f = 2+1+1$ determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	Action	Description
χQCD 14	[48]	2+1	Overlap on DW	$0.29 < am_c < 0.75$. Heavy-quark discretization errors estimated by including $(am_c)^2$ and $am_c)^4$ terms in the chiral/continuum extrapolation.
HPQCD 12A	[165]	2+1	HISQ	$0.41 < am_c < 0.62$. Heavy-quark discretization errors estimated using different fit ansätze to be $\approx$ 1.2%.
FNAL/MILC 11	[166]	2+1	Fermilab	Discretization errors from charm quark estimated through a combination of Heavy Quark and Symanzik Effective Theories to be around 3% for $f_{D_{(s)}}$ and negligible for the ratio.
PACS-CS 11	[167]	2+1	Tsukuba	$am_c \approx 0.57$. Heavy-quark discretization errors estimated to be at the percent level by power counting.
HPQCD 10A	[168]	2+1	HISQ	$0.193 < am_c < 0.825$. Heavy-quark discretization errors estimated by changing the fit-inputs to be $\approx$ 0.4%.
HPQCD/UKQCD 07	[88]	2+1	HISQ	$0.43 < am_c < 0.85$. Heavy-quark discretization errors estimated from the chiral/continuum fits to be $\approx$ 0.5%.
FNAL/MILC 05	[169]	2+1	Fermilab	Discretization errors from charm quark estimated via heavy-quark power-counting at 4.2% for $f_{D_{(s)}}$ and 0.5% for the ratio.

Table 119: Heavy-quark treatment in $N_f = 2+1$ determinations of the D and D_s meson decay constants.

Collab.	Ref.	N_f	Action	Description
TWQCD 14	[170]	2	DW	$am_c \leq 0.55$. Optimal Domain Wall fermions [171] preserving chiral symmetry.
ALPHA 13B	[50]	2	npSW	$am_c \leq 0.28$. Axial current non-perturbatively improved ($\mathcal{O}(am)$ at 1-loop).
ETM 09, 11A, 13B	[55, 57, 92]	2	tmWil	$0.16 < am_c < 0.23$. $D(a_{\rm min}) \approx 5\%$ in ETM 09.

Table 120: Heavy-quark treatment in $N_f = 2$ determinations of the D and D_s meson decay constants.

B.5.2 $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 10B, 11	[172, 173]	2+1	0.09, 0.12	Modified z-expansion fit combining the continuum and chiral extrapolations and the momentum transfer dependence. Leading discretization errors from $(am_c)^n$ charm-mass effects (see Table 125). Subleading $(aE)^n$ discretization corrections estimated to be 1.0% for both $D \to \pi$ and $D \to K$.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 set from several quantities including f_π, f_K, and Υ $2S-1S$ splitting c.f. HPQCD 09B [10]. Scale uncertainty estimated to be 0.7% in $D \to \pi$ and and 0.2% in $D \to K$.
FNAL/MILC 04	[174]	2+1	0.12	Discretization effects from light-quark sector estimated to be 4% by power counting. Discretization effects from final-state pion and kaon energies estimated to be 5%.	Scale set through Υ $2S-1S$ splitting c.f. HPQCD 03 [175]. Error in a^{-1} estimated to be 1.2%, but scale error in dimensionless form factor negligible compared to other uncertainties.
ETM 11B	[176]	2	0.068, 0.086, 0.102	Discretization errors estimated to be 5% for $D \to \pi$ and 3% for $D \to K$ from comparison of results in the continuum limit to those at the finest lattice spacing.	Scale set through f_π c.f. ETM 07A [177] and ETM 09C [128].

Table 121: Continuum extrapolations/estimation of lattice artifacts in determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
HPQCD 10B, 11	[172, 173]	2+1	390, 390	Modified z-expansion fit combining the continuum and chiral extrapolations and the momentum transfer dependence. Contributions to error budget from light valence and sea-quark mass dependence estimated to be 2.0% for $D \to \pi$ and 1.0% for $D \to K$.
FNAL/MILC 04	[174]	2+1	510	Fit to SχPT, combined with the Becirevic-Kaidalov ansatz for the momentum transfer dependence of form factors. Error estimated to be 3% for $D \to \pi$ and 2% for $D \to K$ by comparing fits with and without one extra analytic term.
ETM 11B	[176]	2	270	$SU(2)$ tmHMχPT plus Becirevic-Kaidalov ansatz for fits to the momentum transfer dependence of form factors. Fit uncertainty estimated to be 7% for $D \to \pi$ and 5% for $D \to K$ by considering fits with and without NNLO corrections of order $\mathcal{O}(m_\pi^4)$ and/or higher-order terms through E^5, and by excluding data with $E \gtrsim 1$ GeV.

Table 122: Chiral extrapolation/minimum pion mass in determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\mathrm{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$	Description
HPQCD 10B, 11	[172, 173]	2+1	2.4, 2.4/2.9	$\gtrsim 3.8$	Finite volume effects estimated to be 0.04% for $D \to \pi$ and 0.01% for $D \to K$ by comparing the "$m_\pi^2 \log(m_\pi^2)$" term in infinite and finite volume.
FNAL/MILC 04	[174]	2+1	2.4/2.9	$\gtrsim 3.8$	No explicit estimate of FV error, but expected to be small for simulation masses and volumes.
ETM 11B	[176]	2	2.2, 2.1/2.8, 2.4	$\gtrsim 3.7$	Finite volume uncertainty estimated to be at most 2% by considering fits with and without the lightest pion mass point at $m_\pi L \approx 3.7$.

Table 123: Finite volume effects in determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
HPQCD 10B, 11	[172, 173]	2+1	—	Form factor extracted from absolutely normalized scalar-current matrix element then using kinematic constraint at zero momentum-transfer $f_+(0) = f_0(0)$.
FNAL/MILC 04	[174]	2+1	mNPR	Size of two-loop correction to current renormalization factor assumed to be negligible.
ETM 11B	[176]	2	—	Form factors extracted from double ratios insensitive to current normalization.

Table 124: Operator renormalization in determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors.

Collab.	Ref.	N_f	Action	Description
HPQCD 10B, 11	[172, 173]	2+1	HISQ	Bare charm-quark mass $am_c \sim 0.41$–0.63. Errors of $(am_c)^n$ estimated within modified z-expansion to be 1.4% for $D \to K$ and 2.0% for $D \to \pi$. Consistent with expected size of dominant one-loop cutoff effects on the finest lattice spacing, $\mathcal{O}(\alpha_S(am_c)^2(v/c)) \sim 1.6\%$.
FNAL/MILC 04	[174]	2+1	Fermilab	Discretization errors from charm quark estimated via heavy-quark power-counting to be 7%.
ETM 11B	[176]	2	tmWil	Bare charm-quark mass $am_c \sim 0.17$–0.30. Expected size of $\mathcal{O}((am_c)^2)$ cutoff effects on the finest lattice spacing consistent with quoted 5% continuum-extrapolation uncertainty.

Table 125: Heavy quark treatment in determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors.

B.6 Notes to section 8 on B-meson decay constants, mixing parameters and form factors

In the following, we summarize the characteristics (lattice actions, pion masses, lattice spacings, etc.) of the recent $N_f = 2+1+1$, $N_f = 2+1$ and $N_f = 2$ runs. We also provide brief descriptions of how systematic errors are estimated by the various authors. We focus on calculations with either preliminary or published quantitative results.

B.6.1 $B_{(s)}$-meson decay constants

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ETM 13E	[178]	2+1+1	245, 239, 211	$M_{\pi,\text{min}}$ refers to the charged pions. Linear and NLO (full QCD) HMχPT supplemented by an a^2 term is used for the $SU(3)$ breaking ratios.The chiral fit error is estimated from the difference between the NLO HMχPT and linear fits with half the difference used as estimate of the systematic error. The ratio z_s is fit using just linear HMχPT supplemented by an a^2 term.
HPQCD 13	[179]	2+1+1	310, 294, 173	Two or three pion masses at each lattice spacing, one each with a physical mass GB pion. NLO (full QCD) HMχPT supplemented by generic a^2 and a^4 terms is used to interpolate to the physical pion mass.

Table 126: Chiral extrapolation/minimum pion mass in determinations of the B and B_s meson decay constants for $N_f = 2+1+1$ simulations. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
RBC/UKQCD 14 RBC/UKQCD 13A	[180] [181]	2+1	329, 289	Two or three light quark masses per lattice spacing. In RBC/UKQCD 14, three to four light valence-quark masses that are heavier than the sea-quark masses are also employed to have partially-quenched points. NLO $SU(2)$ HMχPT is used. In RBC/UKQCD 14, the fit with only the unitary points is the central analysis procedure, and the systematic errors in the conbined chiral-continuum extrapolation are estimated to be from 3.1% to 5.9% in the decay constants and the $SU(3)$ breaking ratios.
RBC/UKQCD 14A	[182]	2+1	327, 289	Two or three light quark masses per lattice spacing. NLO $SU(2)$ HMχPT is used in the combined chiral-continuum extrapolation. The systematic errors in this extrapolation are estimated to be 3.54% for f_B, 1.98% for f_{B_s}, and 2.66% for f_{B_s}/f_B.
HPQCD 12	[183]	2+1	390, 390	Two or three pion masses at each lattice spacing. NLO (full QCD) HMχPT supplemented by NNLO analytic terms and generic a^2 and a^4 terms is used. The systematic error is estimated by varying the fit Ansatz, in particular for the NNLO analytic terms and the a^{2n} terms.
HPQCD 11A	[184]	2+1	570, 450, 390, 330, 330	One light sea quark mass only at each lattice spacing. The sea-quark mass dependence is assumed to be negligible, based on the calculation of f_{D_s} in Ref. [168], where the sea quark extrapolation error is estimated as 0.34%.
FNAL/MILC 11	[166]	2+1	570, 440, 320	Three to five sea-quark masses per lattice spacing, and $9-12$ valence light quark masses per ensemble. NLO partially quenched HMrSχPT including $1/m$ terms and supplemented by NNLO analytic and $\alpha_s^2 a^2$ terms is used. The systematic error is estimated by varying the fit Ansatz, in particular the NNLO analytic terms and the chiral scale.
RBC/UKQCD 10C	[185]	2+1	430	Three light quark masses at one lattice spacing. NLO $SU(2)$ χPT is used. The systematic error is estimated from the difference between NLO χPT and linear fits as $\sim 7\%$.
HPQCD 09	[186]	2+1	440, 400	Four or two pion masses per lattice spacing. NLO (full QCD) HMrSχPT supplemented by NNLO analytic terms and $\alpha_s a^2, a^4$ terms is used. The chiral fit error is estimated by varying the fit Ansatz, in particular, by adding or removing NNLO and discretization terms.

Table 127: Chiral extrapolation/minimum pion mass in determinations of the B and B_s meson decay constants for $N_f = 2+1$ simulations. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ALPHA 14 ALPHA 13 ALPHA 12A	[187] [188] [189]	2	280, 190, 270	LO and NLO HMχPT supplemented by a term linear in a^2 are used. In ALPHA 13 and ALPHA 12A, the final result is an average between LO and NLO with half the difference used as estimate of the systematic error. In ALPHA 14, the NLO fit is used as the central analysis procedure, and the LO results are used to estimate the systematic errors (0.9% MeV for f_{B_s}, 1.1% for f_B, and 1.6% for f_{B_s}/f_B).
ETM 13B, 13C ETM 12B ETM 11A	[55, 190] [191] [57]	2	410, 275, 300, 270	$M_{\pi,\text{min}}$ refers to the charged pions. Linear and NLO (full QCD) HMχPT supplemented by an a^2 term is used.The chiral fit error is estimated from the difference between the NLO HMχPT and linear fits with half the difference used as estimate of the systematic error. For the static limit calculation in ETM 11A, Φ_s^{stat} is extrapolated assuming a constant in light quark mass. The ratio $\Phi_s^{\text{stat}}/\Phi_\ell^{\text{stat}}$ is fit using three different chiral fit forms (NLO HMχPT, linear, and quadratic) to estimate the chiral fir error.
ALPHA 11	[192]	2	331, 268, 267	Linear and NLO (full QCD) HMχPT supplemented by a term linear in a^2 are used. The final result is an average between linear and NLO fits with half the difference used as estimate of the systematic error.
ETM 09D	[193]	2	410, 275, 300	$M_{\pi,\text{min}}$ refers to the charged pions. Linear and NLO (full QCD) HMχPT is used. The final result given by the average of NLO HMChiPT and linear *Ansätze* $\pm$ half the difference).

Table 128: Chiral extrapolation/minimum pion mass in determinations of the B and B_s meson decay constantsfor $N_f = 2$ simulations. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 13E	[178]	2+1+1	2.84/2.13, 2.61/1.96, 2.97	3.5, 3.2, 3.2	FV error estimated how?
HPQCD 13	[179]	2+1+1	2.4/3.5/4.7, 2.9/3.8/5.8, 2.8/5.6	7.4, 8.6, 4.9	The analysis uses finite-volume χPT.
RBC/UKQCD 14 RBC/UKQCD 13A	[180] [181]	2+1	2.64, 2.75	4.4, 4.0	In RBC/UKQCD 14, finite-volume effects are estimated to be negligible for f_{B_s}, 0.4% for f_{B^0}, 0.5% for f_{B^+} and the $SU(3)$ breaking ratios.
RBC/UKQCD 14A	[182]	2+1	2.74, 2.76	4.5, 4.0	Finite-volume effects are estimated to be neglegible for f_{B_s}, 0.82% for f_B, and 1% for f_{B_s}/f_B.
HPQCD 12	[183]	2+1	2.4/2.9, 2.5/3.6	5.7, 7.1	FV error is taken from Ref. [88] for HPQCD's D meson analysis, where it was estimated using finite volume χPT .
HPQCD 11A	[184]	2+1	2.4, 2.4, 2.5, 2.9, 2.9	6.9, 5.5, 4.9, 4.8, 4.8	FV error is assumed to negligible.
FNAL/MILC 11	[166]	2+1	2.4, 2.4/2.9, 2.5/3.6	6.9, 6.4, 5.8	FV error is estimated using finite-volume χPT.
RBC/UKQCD 10C	[185]	2+1	1.8	3.9	FV error estimated using finite-volume χPT to be 1% for $SU(3)$ breaking ratios.
HPQCD 09	[186]	2+1	2.4/2.9, 2.5	6.5, 5.1	FV error is assumed to negligible.
ALPHA 14 ALPHA 13 ALPHA 12A ALPHA 11	[187] [188] [189] [192]	2	2.4/3.6, 2.1/3.1/4.2, 2.3/3.1	5.2, 4.1, 4.2	No explicit estimate of FV errors, but expected to be much smaller than other uncertainties.
ETM 13B, 13C ETM 12B ETM 11A	[55, 190] [191] [57]	2	2.4, 2.0/2.7, 2.1, 1.7/2.6	5.0, 3.7, 3.3, 3.5	FV errors are found to be negligible by comparing results at $m_\pi L = 3.3$ and $m_\pi L = 4.3$ for $m_\pi \simeq 310$ MeV.

Table 129: Finite volume effects in determinations of the B and B_s meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
ETM 13E	[178]	2+1+1	0.89, 0.82, 0.62	Combined continuum and chiral extrapolation, linear in a^2.	Scale set from f_π. Scale setting uncertainty included in combined statistical and systematic error.
HPQCD 13	[179]	2+1+1	0.15, 0.12, 0.09	Combined continuum and chiral extrapolation. Continuum extrapolation errors estimated to be 0.7%.	Scale set from Υ(2S-1S) splitting, see Ref. [194]. Scale uncertainty included in statistical error.

Table 130: Continuum extrapolations/estimation of lattice artifacts in determinations of the B and B_s meson decay constants for $N_f = 2+1+1$ simulations.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
RBC/UKQCD 14 RBC/UKQCD 13A	[180] [181]	2+1	0.11, 0.086	Combined continuum and chiral extrapolation with linear in a^2 term. In RBC/UKQCD 14, the systematic errors from this procedure are estimated to be from 3.1% to 5.9% in the decay constants and the $SU(3)$-breaking ratios.	Scale set by the Ω baryon mass. In RBC/UKQCD 14, scale uncertainty estimated to be 1.5% in the decay constants, and 0.1% in the $SU(3)$-breaking ratios.
RBC/UKQCD 14A	[182]	2+1	0.11, 0.086	Chiral-continuum extrapolation with linear in a^2 term is employed, with the systematic errors estimated to be from 1.98% to 3.54% in the decay constants and f_{B_s}/f_B. Discretization errors at $\mathcal{O}(\alpha_s a)$ in the static-light system are estimated to be 1% in the decay constants, and 0.2% in f_{B_s}/f_B.	Scale set by the Ω baryon mass.
HPQCD 12	[183]	2+1	0.12, 0.09	Combined continuum and chiral extrapolation. Continuum extrapolation errors estimated to be 0.9%.	Relative scale r_1/a from the static-quark potential. Absolute scale r_1 from f_π, f_K, and Υ(2S-1S) splitting. Scale uncertainty estimated to be 1.1%.
HPQCD 11A	[184]	2+1	0.15, 0.12, 0.09, 0.06, 0.045	$am_Q \approx 0.2 - 0.85$. Combined continuum and HQET fit. Continuum extrapolation error estimated by varying the fit ansatz and the included data points to be 0.63%. Discretization errors appear to decrease with increasing heavy-meson mass.	Relative scale r_1/a from the static-quark potential. Absolute scale r_1 from f_π, f_K, and Υ(2S-1S) splitting. Scale uncertainty estimated to be 0.74%.
FNAL/MILC 11	[166]	2+1	0.15, 0.12, 0.09	Combined continuum and chiral extrapolation. Continuum extrapolation errors estimated to be 1.3%.	Relative scale r_1/a from the static-quark potential. Absolute scale r_1 from f_π, f_K, and Υ(2S-1S) splitting. Scale uncertainty estimated to be 1 MeV.
RBC/UKQCD 10C	[185]	2+1	0.11	One lattice spacing with discretization errors estimated by power counting as 3%.	Scale set by the Ω baryon mass. Combined scale and mass tuning uncertainties on f_{B_s}/f_B estimated as 1%
HPQCD 09	[186]	2+1	0.12, 0.09	Combined continuum and chiral extrapolation. Continuum extrapolation errors estimated to be 3%.	Relative scale r_1/a from the static-quark potential. Absolute scale r_1 from the Υ(2S-1S) splitting. Scale uncertainty estimated to be 2.3%.

Table 131: Continuum extrapolations/estimation of lattice artifacts in determinations of the B and B_s meson decay constants for $N_f = 2 + 1$ simulations.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
ALPHA 14 ALPHA 13 ALPHA 12A ALPHA 11	[187] [188] [189] [192]	2	0.075, 0.065, 0.048	Combined continuum and chiral extrapolation with linear in a^2 term. Continuum extrapolation errors estimated to be 5 MeV in ALPHA 11. The continuum extrapolation with a term linear in a also inverstigated in ALPHA 14, and within the statistical error no discernable difference was observed.	Relative scale set from r_0. Absolute scale set from f_K. Scale setting uncertainty included in combined statistical and extrapolation error.
ETM 13B, 13C ETM 12B ETM 11A	[55, 190] [191] [57]	2	0.098, 0.085, 0.067, 0.054	Combined continuum and chiral extrapolation, with a term linear in a^2. ETM 12 and 13 include a heavier masses than ETM 11A. Discretization error included in combined statistical and systematic error, estimated by dropping the data at the coarsest lattice spacing as $\sim 0.5-1\%$.	Scale set from f_π. Scale setting uncertainty included in combined statistical and systematic error.
ETM 09D	[193]	2	0.098, 0.085, 0.067	Combined continuum and chiral extrapolation with a term linear in a^2.	Scale set from f_π. Scale setting uncertainty included in combined statistical and systematic error.

Table 132: Continuum extrapolations/estimation of lattice artifacts in determinations of the B and B_s meson decay constants for $N_f = 2$ simulations.

Collab.	Ref.	N_f	Ren.	Description
ETM 13E	[178]	2+1+1	–, PT1ℓ	The current used for the relativistic decay constants is absolutely normalized. The ratio is constructed from the relativistic decay constant data and the heavy-quark pole masses. Ratios of pole-to-$\overline{\rm MS}$ mass conversion factors are included at NLO in continuum perturbation theory.
HPQCD 13	[179]	2+1+1	PT1ℓ	The NRQD effective current is matched through $\mathcal{O}(1/m)$ and renormalized using one-loop PT. Included are all terms though $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s\, a)$, $\mathcal{O}(\Lambda_{\rm QCD}/M)$, $\mathcal{O}(\alpha_s/aM)$, $\mathcal{O}(\alpha_s\, \Lambda_{\rm QCD}/M)$. The dominant error is due unknown $\mathcal{O}(\alpha_s^2)$ contributions to the current renormalization. The perturbation theory used in this work is the same as in HPQCD 09 and 12, but is rearranged to match the mNPR method. Using the fact that the heavy-heavy temporal vector current is normalized, and that the light-light HISQ vector current receives a small one-loop correction, the error is estimated as $\sim 1.4\%$.

Table 133: Description of the renormalization/matching procedure adopted in the determinations of the B and B_s meson decay constants for $N_f = 2+1+1$ simulations.

Collab.	Ref.	N_f	Ren.	Description
RBC/UKQCD 14 RBC/UKQCD 13A	[180] [181]	2+1	mNPR	In RBC/UKQCD 14, the error is dominated by the perturbative aspect, and is estimated to be 1.7% for the decay constants by taking the full size of the one-loop correction for the fine lattice.
RBC/UKQCD 14A	[182]	2+1	PT1ℓ	A two-step matching procedure is employed, first from QCD to HQET in the continuum at m_b, then to HQET on the lattice at a^{-1} with $\mathcal{O}(pa)$ and $\mathcal{O}(m_q a)$ errors included. Both matching steps are accurate to one-loop, and the running between m_b and a^{-1} is performed at two-loop accordingly. The error is estimated using a power-counting argument to be 6% for the decay constants.
HPQCD 12/09	[183, 186]	2+1	PT1ℓ	The NRQD effective current is matched through $\mathcal{O}(1/m)$ and renormalized using one-loop PT. Included are all terms though $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s\, a)$, $\mathcal{O}(\Lambda_{\rm QCD}/M)$, $\mathcal{O}(\alpha_s/aM)$, $\mathcal{O}(\alpha_s\, \Lambda_{\rm QCD}/M)$. The dominant error is due unknown $\mathcal{O}(\alpha_s^2)$ contributions to the current renormalization. The authors take the perturbative error as $\sim 2\rho_0\, \alpha_s^2$, where ρ_0 is the coefficient of the one-loop correction to the leading term, which yields an error of $\sim 4\%$.
HPQCD 11A	[184]	2+1	–	This work uses PCAC together with an absolutely normalized current.
FNAL/MILC 11	[166]	2+1	mNPR	The authors' estimate of the perturbative errors is comparable in size to the actual one-loop corrections.
RBC/UKQCD 10C	[185]	2+1	PT1ℓ	The static-light current is matched through $\mathcal{O}(\alpha_s a, \alpha_s)$ and renormalized using one-loop tadpole improved PT. For massless light quarks, the renormalization factors cancel in the ratio of decay constants.

Table 134: Description of the renormalization/matching procedure adopted in the determinations of the B and B_s meson decay constants for $N_f = 2+1$ simulations.

Collab.	Ref.	N_f	Ren.	Description
ALPHA 14 ALPHA 13 ALPHA 12A ALPHA 11	[187] [188] [189] [192]	2	NPR	The authors use the Schrödinger functional for the NP matching.
ETM 13B, 13C ETM 12B ETM 11A	[55, 190] [191] [57]	2	–, PT1ℓ	The current used for the relativistic decay constants is absolutely normalized. **Interpolation method:** The static limit current renormalization is calculated in one-loop mean field improved perturbation theory, there half the correction is used to estimate the error. **Ratio method:** The ratio is constructed from the relativistic decay constant data and the heavy-quark pole masses. Ratios of pole-to-$\overline{\rm MS}$ mass conversion factors are included at NLO in continuum perturbation theory.

Table 135: Description of the renormalization/matching procedure adopted in the determinations of the B and B_s meson decay constants for $N_f = 2$ simulations.

Collab.	Ref.	N_f	Action	Description
ETM 13E	[178]	2+1+1	tmWil	The estimate of the discretization effects is described in the continuum table. The relativistic data are matched to HQET using NLO continuum PT in an intermediate step, and converted back to QCD at the end. The error due to HQET matching (estimated by replacing the NLO expressions with LO) is a very small contribution to the systematic error due to the heavy quark mass dependence.
HPQCD 13	[179]	2+1+1	NRQCD	The leading HQ truncation effects are of $\mathcal{O}(\Lambda^2_{\rm QCD}/m_h^2)$ and $\mathcal{O}(\alpha_s^2\Lambda_{\rm QCD}/m_h)$, and the errors are at the subpercentage level.

Table 136: Heavy quark treatment in $N_f = 2+1+1$ determinations of the B and B_s meson decay constants.

Collab.	Ref.	N_f	Action	Description
RBC/UKQCD 14 RBC/UKQCD 13A	[180] [181]	2+1	RHQ	In RBC/UKQCD 14, the heavy-quark discretization errors are estimated to be 1.7% in the decay constants, and 0.3% in the $SU(3)$ breaking ratios.
RBC/UKQCD 14A	[182]	2+1	Static	Static-limit computation, with $\mathcal{O}(\Lambda_{\rm QCD}/m_h)$ errors estimated to be 10% for the decay constants, and 2.2% for f_{B_s}/f_B.
HPQCD 12	[183]	2+1	NRQCD	HQ truncation effects estimated as in HPQCD 09 to be 1.0%
HPQCD 11A	[184]	2+1	HISQ	The analysis uses a combined continuum and $1/m$ extrapolation.
FNAL/MILC 11	[166]	2+1	Fermilab	HQ discretization effects are included in the combined chiral and continuum fits, and are estimated by varying the fit Ansatz and excluding the data at the coarsest lattice spacing to be $\sim 2\%$, consistent with simple power counting estimates but larger than the residual discretization errors observed in the data.
RBC/UKQCD 10C	[185]	2+1	Static	Truncation effects of $\mathcal{O}(1/m_h)$ on the $SU(3)$ breaking ratios are estimated by power counting to be 2%.
HPQCD 09	[186]	2+1	NRQCD	The leading HQ truncation effects are of $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_h)$ due to the tree-level coefficient of the $\boldsymbol{\sigma}\cdot\mathbf{B}$ term. The error is estimated by calculating the B^*-B hyperfine splitting and comparing with experiment as 1%.
ALPHA 14 ALPHA 13 ALPHA 12A ALPHA 11	[187] [188] [189] [192]	2	HQET	NP improved through $\mathcal{O}(1/m_h)$. Truncation errors of $\mathcal{O}\left[(\Lambda_{\rm QCD}/m_h)^2\right]$ are not included.
ETM 13B, 13C ETM 12B ETM 11A	[55, 190] [191] [57]	2	tmWil	The estimate of the discretization effects is described in the continuum table. In both methods the relativistic data are matched to HQET using NLO continuum PT in an intermediate step, and converted back to QCD at the end. The error due to HQET matching (estimated by replacing the NLO expressions with LO) is a very small contribution to the systematic error due to the heavy quark mass dependence. The variation observed from adding heavier masses to their data and/or including $1/m_h^3$ terms is $0.4-1.3\%$.

Table 137: Heavy quark treatment in $N_f = 2+1$ and $N_f = 2$ determinations of the B and B_s meson decay constants.

B.6.2 $B_{(s)}$-meson mixing matrix elements

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 16nty	[195]	2+1	0.12, 0.09, 0.06, 0.045	Combined continuum and chiral extrapolation with NLO HMrSχPT and NNLO analytic terms as well as the terms for the heavy quark discretization errors up to a^3, heavy quark mass mismatch, and renormalization error of α_s^2.	Relative scale r_1/a is set via static-quark potential. Absolute scale is set as r_1 = 0.3117(22) fm. See the description of FNAL/MILC 12 below. The scale uncertainty on ξ e.g. is estimated as 0.6%.
RBC/UKQCD 14A	[182]	2+1	0.11, 0.086	Combined continuum and chiral extrapolation with $SU(2)$ NLO HMχPT and linear in quark mass both with $\mathcal{O}(a^2)$ terms. The combined continuum and chiral extrapolation uncertainty is estimated as 2.55, 2.13 and 3.08% for $f_B\sqrt{B_B}$, $f_{B_s}\sqrt{B_{B_s}}$ and ξ respectively.	Scale is set using the Ω^- mass as input [14]. The scale uncertainty is estimated as 1.84, 1.86 and 0.05% for $f_B\sqrt{B_B}$, $f_{B_s}\sqrt{B_{B_s}}$ and ξ respectively.
FNAL/MILC 12	[196]	2+1	0.12, 0.09	Combined continuum and chiral extrapolation with NLO rHMSχPT, NNLO analytic and generic $\mathcal{O}(\alpha_s^2 a^2, a^4)$ terms. Combined statistical, chiral and light-quark discretization error is estimated, by examining the variation with different fit Ansätze to be 3.7% on ξ.	Relative scale r_1/a is set via static-quark potential. Absolute scale $r_1 = 0.3117(22)$ fm is determined [166] through averaging the f_π input and the estimate of HPQCD collaboration [10]. The scale uncertainty on ξ is estimated as 0.2%.
FNAL/MILC 11A	[197]	2+1	0.12, 0.09, 0.06	Combined continuum and chiral extrapolation with NLO rHMSχPT, NNLO analytic and and generic $\mathcal{O}(\alpha_s^2 a^2, a^4)$ terms.	See above. The error in r_1 yields a 3% uncertainty on $f_B^2 B_B$.
RBC/UKQCD 10C	[185]	2+1	0.11	Only one lattice spacing is used. Discretization error is estimated to be 4% on ξ by power counting.	Scale is set using the Ω^- mass as input [22].The error on ξ due to the combined scale and light quark mass uncertainties is estimated as 1%.
HPQCD 09	[186]	2+1	0.12, 0.09	Combined continuum and chiral extrapolation with NLO rHMSχPT and NNLO analytic terms. Light-quark discretization error is estimated as 3, 2 and 0.3% for $f_B\sqrt{B_B}$, $f_{B_s}\sqrt{B_{B_s}}$ and ξ respectively.	Relative scale r_1/a is set via static-quark potential. Absolute scale r_1 = 0.321(5) fm is determined through Υ mass [198]. The error on $f_B\sqrt{B_B}$ due to the scale uncertainty is estimated as 2.3%.
HPQCD 06A	[199]	2+1	0.12	Only one lattice spacing is used. Light-quark discretization error on $f_{B_s}^2 B_{B_s}$ is estimated as 4% by power counting.	Scale is set using the Υ $2S-1S$ splitting as input [198]. The error on $f_B^2 B_B$ due to the scale uncertainty is estimated as 5%.

Table 138: Continuum extrapolations/estimation of lattice artifacts in determinations of the neutral B-meson mixing matrix elements for $N_f = 2+1$ simulations.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
ETM 13B	[55]	2	0.098, 0.085, 0.067, 0.054	Combined chiral and continuum extrapolation, with a term linear in a^2. Discretization error is estimated by omitting the coarsest lattice as 0.5, 1.7, 1.3 and 1.0 % for B_{B_s}, B_B, B_{B_s}/B_B and ξ respectively. The heavy-quark masses vary in the range $0.13 \lesssim am_h \lesssim 0.85$.	See below.
ETM 12A, 12B	[191, 200]	2	0.098, 0.085, 0.067	Combined chiral and continuum extrapolation, with a term linear in a^2. Discretization error included in combined statistical, chiral and continuum extrapolation error and estimated as 4.5%. The heavy-quark masses vary in the range $0.25 \lesssim am_h \lesssim 0.6$.	Relative scale r_0/a set from the static-quark potential. Absolute scale set from f_π. Scale setting uncertainty included in combined statistical and systematic error.

Table 139: Continuum extrapolations/estimation of lattice artifacts in determinations of the neutral B-meson mixing matrix elements for $N_f = 2$ simulations.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
FNAL/MILC 16nty	[195]	2+1	464, 280, 257, 332	Combined continuum and chiral extrapolation with NLO HMrSχPT, NNLO analytic terms and other discretization errors. See the entry in Table 138. The breakdown of the chiral error on ξ is 0.4% and not the dominant one.
RBC/UKQCD 14A	[182]	2+1	327, 289	Combined continuum and chiral extrapolation with $SU(2)$ NLO HMχPT and linear in quark mass both with $\mathcal{O}(a^2)$ terms. The chiral fit error is estimated from difference between the NLO HMχPT and linear fits, and further from eliminating the heaviest ud quark mass point.
FNAL/MILC 12	[196]	2+1	440, 320	Combined continuum and chiral extrapolation with NLO rHMSχPT and NNLO analytic terms. See the entry in Table 138. The omission of wrong-spin contributions [201] in the HMrSχPT is treated as a systematic error and estimated to be 3.2% for ξ.
FNAL/MILC 11A	[197]	2+1	440, 320, 250	Combined continuum and chiral extrapolation with NLO rHMSχPT and NNLO analytic terms.
RBC/UKQCD 10C	[185]	2+1	430	Linear fit matched with $SU(2)$ NLO HMχPT at the lightest ud mass point is used as the preferred fit. Many different fit Ansätze are considered. The systematic error is estimated from the difference between the $SU(2)$ HMχPT fit described above and a linear fit.
HPQCD 09	[186]	2+1	440, 400	Combined continuum and chiral extrapolation with NLO rHMSχPT and NNLO analytic terms.
HPQCD 06A	[199]	2+1	510	Two sea ud quark masses $m_{ud}/m_s = 0.25$ and 0.5 are used to calculate the matrix element for B_s meson at the predetermined value of the strange quark mass. No significant sea quark mass dependence is observed and the value at the lighter sea ud mass is taken as the result.
ETM 13B ETM 12A,12B	[55] [191, 200]	2	410, 275, 300, 270	$M_{\pi,\text{min}}$ refers to the charged pions, where 270 MeV on the finest lattice only included in ETM 13B. Linear and NLO (full QCD) HMχPT supplemented by an a^2 term is used. The chiral fit error is estimated from the difference between the NLO HMχPT and linear fits.

Table 140: Chiral extrapolation/minimum pion mass in determinations of the neutral B-meson mixing matrix elements. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 16nty	[195]	2+1	2.4/2.9, 2.5/2.9/3.6/5.8, 2.9/3.4/3.8, 2.9	6.8, 8.2, 5.0, 4.8	FV error is estimated to be less than 0.1% for $SU(3)$ breaking ratios from FV HMrSχPT.
RBC/UKQCD 14A	[182]	2+1	2.74, 2.76	4.5, 4.0	FV error is estimated from $SU(2)$ χPT to be 0.76, 0, 1.07% for $f_B\sqrt{B_B}$, $f_{B_s}\sqrt{B_{B_s}}$ and ξ respectively.
FNAL/MILC 12	[196]	2+1	2.4/2.9, 2.5	6.4, 5.1	FV error is estimated to be less than 0.1% for $SU(3)$ breaking ratios from FV HMrSχPT.
FNAL/MILC 11A	[197]	2+1	2.4/2.9, 2.5/2.9/3.6, 3.8	6.4, 5.8, 4.9	FV error on $f_B\sqrt{B_B}$ is estimated to be less than 1%, which is inferred from the study of the B-meson decay constant using FV HMχPT [166].
RBC/UKQCD 10C	[185]	2+1	1.8	3.9	FV error estimated through FV HMχPT as 1% for $SU(3)$ breaking ratios.
HPQCD 09	[186]	2+1	2.4/2.9, 2.5	6.4, 5.1	No explicit estimate of FV error, but expected to be much smaller than other uncertainties.
HPQCD 06A	[199]	2+1	2.4	6.2	No explicit estimate of FV error, but expected to be much smaller than other uncertainties.
ETM 13B ETM 12A,12B	[55] [191, 200]	2	2.4, 2.0/2.7, 2.1, 1.7/2.6	5.0, 3.7, 3.3, 3.5	$L = 1.7/2.6$ fm only included in ETM 13B. FV error is assumed to be negligible based on the study of D-meson decay constants in Ref. [92].

Table 141: Finite volume effects in determinations of the neutral B-meson mixing matrix elements. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available).

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 16nty	[195]	2+1	mNPR	mNPR is used with one-loop lattice perturbation theory to renormalize the four-quark operators with heavy quarks rotated to eliminate tree-level $\mathcal{O}(a)$ errors. The error from neglecting higher order corrections is estimated to be 0.5% on ξ.
RBC/UKQCD 14A	[182]	2+1	PT1l	Static-light four-quark operators are renormalized with one-loop mean field improved PT. The errors due to neglected higher order effects are estimated for purely $\mathcal{O}\alpha_s^2$ to be 6% on the matrix elements or 1.2% on ξ and for $\mathcal{O}\alpha_s^2 a^2$ to be 1% or 0.2% respectively.
FNAL/MILC 12	[196]	2+1	PT1l	One-loop mean-field improved PT is used to renormalize the four-quark operators with heavy quarks rotated to eliminate tree-level $\mathcal{O}(a)$ errors. The error from neglecting higher order corrections is estimated to be 0.5% on ξ.
FNAL/MILC 11A	[197]	2+1	PT1l	One-loop mean-field improved PT is used to renormalize the four-quark operators with heavy quarks rotated to eliminate tree-level $\mathcal{O}(a)$ errors. The error from neglected higher order corrections is estimated to be 4% on $f_B\sqrt{B_B}$.
RBC/UKQCD 10C	[185]	2+1	PT1l	Static-light four-quark operators are renormalized with one-loop mean field improved PT. The error due to neglected higher order effects is estimated to be 2.2% on ξ.
HPQCD 09	[186]	2+1	PT1l	Four-quark operators in lattice NRQCD are matched to QCD through order α_s, $\Lambda_{\rm QCD}/M$ and $\alpha_s/(aM)$ [202] using one-loop PT. The error due to neglected higher order effects is estimated to be 4% on $f_B\sqrt{B_B}$ and 0.7% on ξ.
HPQCD 06A	[199]	2+1	PT1l	Four-quark operators in lattice NRQCD are matched to full QCD through order α_s, $\Lambda_{\rm QCD}/M$ and $\alpha_s/(aM)$ [202]. The error is estimated as $\sim 1\cdot\alpha_s^2$ to be 9% on $f_{B_s}^2 B_{B_s}$
ETM 13B, 12A, 12B	[55, 191, 200]	2	NPR	The bag parameters are nonperturbatively renormalized in the RI'-MOM scheme. They are calculated as functions of the ($\overline{\rm MS}$) heavy-quark mass (renormalized nonperturbatively in RI/MOM).

Table 142: Operator renormalization in determinations of the neutral B-meson mixing matrix elements.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 16nty	[195]	2+1	Fermilab	The heavy-quark discretization error is a dominant error comparable to the statistical error. It reads 4.6%, 3.2% or 0.7% for the B_d, B_s matrix element or ξ.
RBC/UKQCD 14A	[182]	2+1	Static	Two different static-quark actions with HYP1 and HYP2 smearings are used and the continuum extrapolation is constrained so the two values converges in the limit. The error due to the missing $1/m_b$ corrections is estimated to be 12% for individual matrix elements or 2.2% on ξ using power-counting.
FNAL/MILC 12	[196]	2+1	Fermilab	The heavy-quark discretization error on ξ is estimated to be 0.3 %. The error on ξ due to the uncertainty in the b-quark mass is estimated to be 0.4 %.
FNAL/MILC 11A	[197]	2+1	Fermilab	The heavy-quark discretization error on $f_B\sqrt{B_B}$ is estimated as 4% using power-counting.
RBC/UKQCD 10C	[185]	2+1	Static	Two different static-quark actions with Ape and HYP smearings are used. The discretization error on ξ is estimated as $\sim$ 4% and the error due to the missing $1/m_b$ corrections as $\sim$ 2%, both using power-counting.
HPQCD 09	[186]	2+1	NRQCD	Heavy-quark truncation errors due to relativistic corrections are estimated to be 2.5, 2.5 and 0.4 % for $f_B\sqrt{B_B}$, $f_{B_s}\sqrt{B_{B_s}}$ and ξ respectively.
HPQCD 06A	[199]	2+1	NRQCD	Heavy-quark truncation errors due to relativistic corrections are estimated to be 3% for $f^2_{B_s}B_{B_s}$.
ETM 13B ETM 12A,12B	[55] [191, 200]	2	tmWil	The ratio method is used to perform an interpolation to the physical b quark mass from the simulated heavy mass and the known static limit. In an intermediate step, the ratios include HQET matching factors calculated to tree-level, leading-log, and next-to-leading-log (ETM 13B only) in continuum PT. The interpolation uses a polynomial up to quadratic in the inverse quark-mass. The systematic errors added together with those of the chiral fit are estimated as $1.3-1.6\%$ for bag parameters for ETM 13B, while they are estimated from changing the interpolating polynomial as 2% and from changing the order of HQET matching factors as 3% for ETM 12A and 12B.

Table 143: Heavy-quark treatment in determinations of the neutral B-meson mixing matrix elements.

B.6.3 Form factors entering determinations of $|V_{ub}|$ ($B \to \pi l \nu$, $B_s \to K l \nu$, $\Lambda_b \to p l \nu$)

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 15	[203]	2+1	0.045, 0.06, 0.09, 0.12	Fit to HMrSχPT to remove light-quark discretization errors. Residual heavy-quark discretization errors estimated with power-counting. Total (stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$) error estimated to be 3.1% for f_+ and 3.8% for f_0 at q^2 = 20 GeV^2.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [166].
Detmold 15 $\Lambda_b \to p$	[204]	2+1	0.0849(12), 0.1119(17)	Joint chiral-continuum extrapolation, combined with fit to q^2 dependence of form factors in a "modified" z-expansion. Systematics estimated by varying fit form and $\mathcal{O}(a)$ improvement parameter values.	Set from $\Upsilon(2S)$–$\Upsilon(1S)$ splitting, cf. [205].
RBC/UKQCD 15	[206]	2+1	0.086,0.11	Joint chiral-continuum extrapolation using $SU(2)$ hard-pion HMχPT. Systematic uncertainty estimated by varying fit ansatz and form of coefficients, as well as implementing different cuts on data; ranges from 5.0% to 10.9% for $B \to \pi$ form factors, and 2.5% to 5.1% for $B_s \to K$. Light-quark and gluon discretization errors estimated at 1.1% and 1.3%, respectively.	Scale implicitly set in the light-quark sector using the Ω^- mass, cf. [14].
HPQCD 14	[207]	2+1	0.09,0.12	Combined chiral-continuum extrapolation using hard-pion rHMSχPT. (No explicit estimate of discretization effects.)	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 set to 0.3133(23) fm.
HPQCD 06	[208]	2+1	0.09,0.12	Central values obtained from data at a = 0.12 fm. Discretization errors observed to be within the statistical error by comparison with data at $a = 0.09$ fm.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 set through Υ $2S-1S$ splitting c.f. HPQCD 05B [198].

Table 144: Continuum extrapolations/estimation of lattice artifacts in determinations of $B \to \pi \ell \nu$, $B_s \to K \ell \nu$, and $\Lambda_b \to p \ell \nu$ form factors.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
FNAL/MILC 15	[203]	2+1	330, 260, 280, 470	Simultaneous chiral-continuum extrapolation and q^2 interpolation using NNLO $SU(2)$ hard-pion HMrSχPT. Systematic error estimated by adding higher-order analytic terms and varying the B^*-B-π coupling.
Detmold 15 $\Lambda_b \to p$	[204]	2+1	227, 245 (valence pions)	Joint chiral-continuum extrapolation, combined with fit to q^2 dependence of form factors in a "modified" z-expansion. Only analytic NLO terms $\propto (m_\pi^2 - m_{\pi,\rm phys}^2)$ included in light mass dependence. Systematic uncertainty estimated by repeating fit with added higher-order terms.
RBC/UKQCD 15	[206]	2+1	289, 329	Joint chiral-continuum extrapolation using $SU(2)$ hard-pion HMχPT. Systematic uncertainty estimated by varying fit ansatz and form of coefficients, as well as implementing different cuts on data; ranges from 5.0% to 10.9% for $B \to \pi$ form factors, and 2.5% to 5.1% for $B_s \to K$.
HPQCD 14	[207]	2+1	295, 260	Combined chiral-continuum extrapolation using hard-pion rHMSχPT. (No explicit estimate of extrapolation systematics.)
HPQCD 06	[208]	2+1	400, 440	First interpolate data at fixed quark mass to fiducial values of E_π using the Becirevic-Kaidalov and Ball-Zwicky ansätze, then extrapolate data at fixed E_π to physical quark masses using $SU(3)$ rHMSχPT. Systematic error estimated by varying interpolation and extrapolation fit functions.

Table 145: Chiral extrapolation/minimum pion mass in determinations of $B \to \pi\ell\nu$, $B_s \to K\ell\nu$, and $\Lambda_b \to p\ell\nu$ form factors. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 15	[203]	2+1	2.9, 2.9/3.4/3.8, 2.5/2.9/3.6/5.8, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated by replacing infinite-volume chiral logs with sums over discrete momenta, found to be negligible.
Detmold 15 $\Lambda_b \to p$	[204]	2+1	2.7, 2.7	$\gtrsim 3.1$ (valence sector)	FV effect estimated at 3% from experience on χPT estimates of FV effects for heavy-baryon axial couplings.
RBC/UKQCD 15	[206]	2+1	2.8, 2.6	$4.0, 4.4$	FV effects estimated by correction to chiral logs due to sums over discrete momenta; quoted 0.3-0.5% for f_+ and 0.4-0.7% for f_0 for $B \to \pi$, and 0.2% for f_+ and 0.1-0.2% for f_0 for $B_s \to K$.
HPQCD 14	[207]	2+1	2.5, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated by shift of pion log, found to be negligible.
HPQCD 06	[208]	2+1	2.4/2.9	$\gtrsim 3.8$	No explicit estimate of FV error, but expected to be much smaller than other uncertainties.

Table 146: Finite volume effects in determinations of $B \to \pi\ell\nu$, $B_s \to K\ell\nu$, and $\Lambda_b \to p\ell\nu$ form factors. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 15	[203]	2+1	mNPR	Perturbative truncation error estimated at 1% with size of 1-loop correction on next-to-finer ensemble.
Detmold 15 $\Lambda_b \to p$	[204]	2+1	mNPR	Perturbative truncation error estimated at 1% with size of 1-loop correction on next-to-finer ensemble.
RBC/UKQCD 15	[206]	2+1	mNPR	Perturbative truncation error estimated as largest of power counting, effect from value of α_s used, numerical integration. Non-perturbative normalization of flavour-diagonal currents computed by fixing values of ratios of meson two-point functions to three-point functions with an extra current inversion, cf. [180]
HPQCD 14	[207]	2+1	mNPR	Currents matched using one-loop HISQ lattice perturbation theory, omitting $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b$. Systematic uncertainty resulting from one-loop matching and neglecting $\mathcal{O}(\Lambda^2_{\rm QCD}/m_b^2$ terms estimated at 4% from power counting.
HPQCD 06	[208]	2+1	PT1ℓ	Currents included through $\mathcal{O}(\alpha_S\Lambda_{\rm QCD}/M$, $\alpha_S/(aM)$, $\alpha_S\, a\Lambda_{\rm QCD})$. Perturbative truncation error estimated from power-counting.

Table 147: Operator renormalization in determinations of $B \to \pi\ell\nu$, $B_s \to K\ell\nu$, and $\Lambda_b \to p\ell\nu$ form factors.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 15	[203]	2+1	Fermilab	Total statistical + chiral extrapolation + heavy-quark discretization + $g_{B^*B\pi}$ error estimated to be 3.1% for f_+ and 3.8% for f_0 at $q^2 = 20$ GeV^2.
Detmold 15 $\Lambda_b \to p$	[204]	2+1	Columbia RHQ	Discretization errors discussed as part of combined chiral-continuum-q^2 fit, stemming from $a^2\|\mathbf{p}\|^2$ terms.
RBC/UKQCD 15	[206]	2+1	Columbia RHQ	Discretization errors estimated by power counting to be 1.8% for f_+ and 1.7% for f_0.
HPQCD 14	[207]	2+1	NRQCD	Currents matched using one-loop HISQ lattice perturbation theory, omitting $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/m_b$. Systematic uncertainty resulting from one-loop matching and neglecting $\mathcal{O}(\Lambda^2_{\rm QCD}/m_b^2$ terms estimated at 4% from power counting.
HPQCD 06	[208]	2+1	NRQCD	Discretization errors in $f_+(q^2)$ estimated to be $\mathcal{O}(\alpha_s(a\Lambda_{\rm QCD})^2) \sim 3\%$. Relativistic errors estimated to be $\mathcal{O}((\Lambda_{\rm QCD}/M)^2) \sim 1\%$.

Table 148: Heavy quark treatment in determinations of $B \to \pi\ell\nu$, $B_s \to K\ell\nu$, and $\Lambda_b \to p\ell\nu$ form factors.

B.6.4 Form factors for $B \to K\ell^+\ell^-$

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 15D	[209]	2+1	0.045, 0.06, 0.09, 0.12	Fit to SU(2) HMrSχPT for the combined chiral-continuum limit extrapolation. Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [166].
HPQCD 13E	[210]	2+1	0.09,0.12	Combined chiral-continuum extrapolation using rHMSχPT. Errors provided as a function of q^2, combined total ranging from $\sim 3\%$ to $\sim 5\%$ in data region.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 set to 0.3133(23) fm.

Table 149: Continuum extrapolations/estimation of lattice artifacts in determinations of form factors for $B \to K\ell^+\ell^-$.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
FNAL/MILC 15D	[209]	2+1	330, 260, 280, 470	Simultaneous chiral-continuum extrapolation and q^2 interpolation using SU(2) HMrSχPT, with a hard-kaon χPT treatment of high-energy kaons. Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.
HPQCD 13E	[210]	2+1	295, 260	Combined chiral-continuum extrapolation using rHMSχPT. Errors provided as a function of q^2, combined total ranging from $\sim 3\%$ to $\sim 5\%$ in data region.

Table 150: Chiral extrapolation/minimum pion mass in determinations of form factors for $B \to K\ell^+\ell^-$. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
FNAL/MILC 15D	[209]	2+1	2.9, 2.9/3.8, 2.5/2.9/3.6/5.8, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated by replacing infinite-volume chiral logs with sums over discrete momenta, found to be negligible.
HPQCD 13E	[210]	2+1	2.5, 2.4/2.9	$\gtrsim 3.8$	FV effects included in combined chiral-continuum extrapolation.

Table 151: Finite volume effects in determinations of form factors for $B \to K\ell^+\ell^-$. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 15D	[209]	2+1	mNPR	Perturbative truncation error estimated at 1% for f_+ and f_0 and 2% for f_T, using size of 1-loop correction on next-to-finer ensemble.
HPQCD 13E	[210]	2+1	mNPR	Currents matched using one-loop massless-HISQ lattice perturbation theory. Associated systematic uncertainty dominates quoted 4% uncertainty from matching, charm quenching, and electromagnetic and isospin breaking effects.

Table 152: Operator renormalization in determinations of form factors for $B \to K\ell^+\ell^-$.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 15D	[209]	2+1	Fermilab	Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.
HPQCD 13E	[210]	2+1	NRQCD	Currents matched using one-loop massless-HISQ lattice perturbation theory. Associated systematic uncertainty dominates quoted 4% uncertainty from matching, charm quenching, and electromagnetic and isospin breaking effects.

Table 153: Heavy quark treatment in determinations of form factors for $B \to K\ell^+\ell^-$.

B.6.5 Form factors entering determinations of $|V_{cb}|$ ($B \to D^* l\nu$, $B \to Dl\nu$, $B_s \to D_s l\nu$, $\Lambda_b \to \Lambda_c l\nu$) and $R(D)$)

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 15	[211]	2+1	0.09, 0.12	Combined chiral-continuum extrapolation as part of modified z-expansion of form factors, which also includes uncertainty related to matching of NRQCD and relativistic currents.	Implicitly set from r_1.
FNAL/MILC 15C	[212]	2+1	0.045, 0.06, 0.09, 0.12	Combined chiral-continuum extrapolation using HMrSχPT. Form factors fitted to NLO χPT, with chiral logs taken from staggered version of the Chow-Wise result, modified to include taste-breaking terms. $\mathcal{O}(a^2)$ terms introduced based on power-counting arguments. Total uncertainty estimated at 0.6% for f_+ and 0.5% for f_0 for the largest recoil.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [166]. Uncertainty related to scale setting estimated at 0.2%.
Detmold 15 $\Lambda_b \to \Lambda_c$	[204]	2+1	0.0849(12), 0.1119(17)	Joint chiral-continuum extrapolation, combined with fit to q^2 dependence of form factors in a "modified" z-expansion. Systematics estimated by varying fit form and $\mathcal{O}(a)$ improvement parameter values.	Set from $\Upsilon(2S)$–$\Upsilon(1S)$ splitting, cf. [205].
FNAL/MILC 14	[213]	2+1	0.045, 0.06, 0.09, 0.12, 0.15	Combined chiral-continuum extrapolation using HMrSχPT. Total uncertainty quoted at 0.5%.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [166]. Uncertainty related to scale setting estimated at 0.1%.
Atoui 13	[214]	2	0.054, 0.067, 0.085, 0.098	Combined continuum and chiral extrapolation, with linear terms in a^2 and m_{sea}. No dependence on a or m_{sea} observed within errors. Stability of results vs fits with no m_{sea} dependence checked.	Scale set through F_π.

Table 154: Continuum extrapolations/estimation of lattice artifacts in determinations of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B_s \to D_s\ell\nu$, and $\Lambda_b \to \Lambda_c\ell\nu$ form factors, and of $R(D)$.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
HPQCD 15	[211]	2+1	295, 260	Combined chiral-continuum extrapolation as part of modified z-expansion of form factors. Hard-pion χPT for light mass dependence used to estimate systematic uncertainty to be 1.14%.
FNAL/MILC 15C	[212]	2+1	330, 260, 280, 470	Combined chiral-continuum extrapolation using HMrSχPT. Form factors fitted to NLO χPT, with chiral logs taken from staggered version of the Chow-Wise result, modified to include taste-breaking terms. $\mathcal{O}(a^2)$ terms introduced based on power-counting arguments. Total uncertainty estimated at 0.6% for f_+ and 0.5% for f_0 for the largest recoil.
Detmold 15 $\Lambda_b \to \Lambda_c$	[204]	2+1	227, 245 (valence pions)	Joint chiral-continuum extrapolation, combined with fit to q^2 dependence of form factors in a "modified" z-expansion. Only analytic NLO terms $\propto (m_\pi^2 - m_{\pi,\text{phys}}^2)$ included in light mass dependence. Systematic uncertainty estimated by repeating fit with added higher-order terms.
FNAL/MILC 14	[213]	2+1	330, 260, 280, 470, 590	Combined chiral-continuum extrapolation using HMrSχPT. Systematic errors estimated by adding higher-order analytic terms and varying the D^*-D-π coupling. Total uncertainty quoted at 0.5%.
Atoui 13	[214]	2	270, 300, 270, 410	Combined continuum and chiral extrapolation, with linear terms in a^2 and m_{sea}. No dependence on a or m_{sea} observed within errors. Stability of results vs fits with no m_{sea} dependence checked.

Table 155: Chiral extrapolation/minimum pion mass in determinations of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B_s \to D_s\ell\nu$, and $\Lambda_b \to \Lambda_c\ell\nu$ form factors, and of $R(D)$. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$	Description
HPQCD 15	[211]	2+1	2.5, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated to be negligible.
FNAL/MILC 15C	[212]	2+1	2.9, 2.9–3.8, 2.5–5.8, 2.4/2.9	$\gtrsim 3.8$	FV error estimated to be negligible in [215].
Detmold 15 $\Lambda_b \to \Lambda_c$	[204]	2+1	2.7, 2.7	$\gtrsim 3.1$ (valence sector)	FV effect estimated at 1.5% from experience on χPT estimates of FV effects for heavy-baryon axial couplings.
FNAL/MILC 14	[213]	2+1	2.9, 2.9–3.8, 2.4–5.5, 2.4/2.9, 2.4	$\gtrsim 3.8$	FV error estimated to be negligible.
Atoui 13	[214]	2	1.7/2.6, 2.1, 2.0/2.7, 2.4	$\gtrsim 3.6$	No volume dependence observed within errors.

Table 156: Finite volume effects in determinations of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B_s \to D_s\ell\nu$, and $\Lambda_b \to \Lambda_c\ell\nu$ form factors, and of $R(D)$. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
HPQCD 15	[211]	2+1	One loop.	One-loop matching of currents taken from [216].
FNAL/MILC 15C	[212]	2+1	mNPR	Form factors extracted from ratios of correlators that renormalize with ratios of current normalizations, computed at one-loop in perturbation theory. Dependence of renormalization factor on recoil parameter w neglected. Systematic uncertainty due to perturbative truncation and w-dependence estimated by power counting to 0.7%.
Detmold 15 $\Lambda_b \to \Lambda_c$	[204]	2+1	mNPR	Perturbative truncation error estimated at 1% with size of 1-loop correction on next-to-finer ensemble.
FNAL/MILC 14	[213]	2+1	mNPR	Majority of current renormalization factor cancels in double ratio of lattice correlation functions. Remaining correction calculated with 1-loop tadpole-improved lattice perturbation theory. Systematic uncertainty estimated at 0.4%.
Atoui 13	[214]	2	—	Observables obtained from ratios that do not require renormalization. Checks performed by comparing with results coming from currents that are renormalized separately with non-perturbative $Z_{\rm V}$.

Table 157: Operator renormalization in determinations of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B_s \to D_s\ell\nu$, and $\Lambda_b \to \Lambda_c\ell\nu$ form factors, and of $R(D)$.

Collab.	Ref.	N_f	Action	Description
HPQCD 15	[211]	2+1	NRQCD for b quark, HISQ for c quark	Discretization errors estimated via power counting to be 2.59%.
FNAL/MILC 15C	[212]	2+1	Fermilab	Discretization errors of form factors estimated via power counting to be 0.4%.
Detmold 15 $\Lambda_b \to \Lambda_c$	[204]	2+1	Columbia RHQ	Discretization errors discussed as part of combined chiral-continuum-q^2 fit, stemming from $a^2\|\mathbf{p}\|^2$ terms.
FNAL/MILC 14	[213]	2+1	Fermilab	Discretization errors estimated via power counting to be 1%.
Atoui 13	[214]	2	tmWil	Results obtained from step-scaling in heavy quark mass via the ratio method. Separate continuum limit extrapolations with mild a^2 dependence carried out for each mass point separately. Result at physical value of m_b obtained by interpolation between data region and known exact HQET limit.

Table 158: Heavy quark treatment in determinations of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B_s \to D_s\ell\nu$, and $\Lambda_b \to \Lambda_c\ell\nu$ form factors, and of $R(D)$.

B.7 Notes to section 9 on the strong coupling α_s

B.7.1 Renormalization scale and perturbative behaviour

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
FlowQCD 15	[217]	0	0.09-0.12	2	$\alpha_{\overline{\rm MS}}(2.63/a)$ computed from the boosted coupling. The physical volume ranges from $2.4 \sim 3.8$ fm.
Sternbeck 12	[218]	0	0.11-0.18	3	$\alpha_{\rm T}(p)$ for $p = 5-40$ GeV. Fitted to PT without power corrections. ($\beta = 6.0$, 6.4, 6.7, 6.92.)
Ilgenfritz 10	[219]	0	0.07-0.9	3	$\alpha_{\rm T}(p)$ for $p = 1-240$ GeV. ($\beta = 5.8$, 6.0, 6.2, 6.4, 9.0.)
Sternbeck 10	[220]	0	0.07-0.32	3	$\alpha_{\rm T}$ for $p = 2.5-140$ GeV, fitted to PT partially on very small lattices.
Brambilla 10	[221]	0	0.22-0.47	3	$\alpha_{\rm qq}(1/r)$ for the range $r/r_0 = 0.15-0.5$. Fit of $V(r)$ to PT with renormalon subtraction and resummation reproduces the static potential for $r/r_0 = 0.15-0.45$ well.
Boucaud 08	[222]	0	0.18-0.35	3	$\alpha_{\rm T}(p)$ with $p = 3-6$ GeV. Fitted to PT with $1/p^2$ correction.
Boucaud 05	[223]	0	0.22-0.55	3	$\Lambda_{\widetilde{\rm MOM}_{g,c}}$ using gluon and ghost propagators with $2 \leq \mu \leq 6$ GeV. Fit to perturbation theory.
QCDSF-UKQCD 05	[224]	0	0.10-0.15	2	$\alpha_{\overline{\rm MS}}(2.63/a)$ computed from the boosted coupling.
CP-PACS 04	[225]	0	0.08-0.28	2	$\alpha_{\rm SF}(1/L)$ step-scaling functions at $\alpha_{\rm eff} = 0.08, 0.19$, study of continuum limit. Agreement of continuum limit with ALPHA 98.
Boucaud 01A	[226]	0	0.18-0.45	2	$\alpha_{\rm MOM}$ with $p = 2.5-10$ GeV. Consistency check of $n_l = 2$ loop perturbation formula with gluon condensate. $\langle A^2\rangle$ from $\alpha_{\rm MOM}$ and gluon propagator are consistent.
Soto 01	[227]	0	0.25-0.36, 0.3-0.36, 0.19-0.24	2	$\alpha_{\widetilde{\rm MOM}}$ for $p = 3-10$ GeV. Fit with $n_l = 2$ loop formula with gluon condensate. (Without condensate does not fit the lattice data.) ($\beta = 6.0$, 6.2, 6.8.)
Boucaud 00A	[228]	0	0.35-0.55, 0.25-0.45, 0.22-0.28, 0.18-0.22	2	$\alpha_{\widetilde{\rm MOM}}$ with $p = 2-10$ GeV. Fitted to $n_l = 2$ loop perturbation theory with power correction. ($\beta = 6.0$, 6.2, 6.4, 6.8.)
Boucaud 00B	[229]	0	0.35-0.55, 0.25-0.45, 0.22-0.28, 0.18-0.22	2	$\alpha_{\rm MOM}$ with $2 \leq \mu \leq 10$ GeV. Consistency check of $n_l = 2$ loop perturbation formula with gluon condensate. $\beta_2^{MOM} = 1.5 \times \beta_2^{\widetilde{MOM}}$ is needed. ($\beta = 6.0$, 6.2, 6.4, 6.8.)

Table 159: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 0$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Becirevic 99A	[230]	0	0.25-0.4	2	$\alpha_{\widetilde{\rm MOM}}$ with $p = 2.5$ - 5.5 GeV.
Becirevic 99B	[231]	0	0.18-0.25	2	$\alpha_{\widetilde{\rm MOM}}$ from a single lattice spacing with $p = 5.6$ - 9.5 GeV.
SESAM 99	[232]	0	0.15	1	$\alpha_{\rm V}(3.41/a)$ computed from the boosted coupling.
ALPHA 98	[233]	0	0.07-0.28	2	$\alpha_{\rm SF}(1/L)$ step scaling, agreement with perturbative running ($n_l = 2$) for $\alpha_{\rm eff} < 0.15$.
Boucaud 98A	[234]	0	0.35-0.5	1,2	$\alpha_{\rm MOM}$, with $2.1 \le \mu \le 3.9$ GeV. $n_l = 1$ for $\alpha_{\rm MOM}$, $n_l = 2$ for $\alpha_{\widetilde{\rm MOM}}$.
Boucaud 98B	[235]	0	0.27-0.50	2	$\alpha_{\widetilde{\rm MOM}}$ with $\mu = 2.2 - 4.5$ GeV.
Alles 96	[236]	0	0.35-0.71	1	$\alpha_{\widetilde{\rm MOM}}(p)$ with $p = 1.8 - 3.0$ GeV.
Wingate 95	[237]	0	0.15	1	$\alpha_V(3.41/a)$ computed from the boosted coupling.
Davies 94	[238]	0	0.15	1	$\alpha_V(3.41/a)$ computed from the boosted coupling.
Lüscher 93	[239]	0	0.09-0.28	1	$\alpha_{\rm SF}(1/L)$ step scaling, agreement with perturbative running ($n_l = 1$) for $\alpha_{\rm eff} < 0.17$.
UKQCD 92	[240]	0	0.17-0.40	1	$\alpha_{\rm qq}(1/r)$ for a single lattice spacing. Fit of $\alpha_{\rm qq}(1/r)$ to a NLO formula.
Bali 92	[241]	0	0.15-0.35	1	$\alpha_{\rm qq}(1/r)$ for the lattice spacing used in the analysis. Box size $L \approx 1.05$ fm. Fit of $\alpha_{\rm qq}(1/r)$ to a NLO formula. $\Lambda_{\overline{\rm MS}}$ is found to depend on the fit-range.
El-Khadra 92	[242]	0	0.12-0.15	1	$\alpha_{\overline{\rm MS}}(\pi/a)$ from 1-loop boosted perturbation theory.

Table 159: (contd.) Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 0$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Karbstein 14	[243]	2	0.28 - 0.41	3	$\alpha_V(p)$ for momentum $1.5 < p < 3.0\,\mathrm{GeV}$. Values computed from the quoted Λ parameter with the 2-loop β function; larger values $(0.32 - 0.62)$ are obtained with 3-loop running. As with ETM 11C central values are taken from $a = 0.042\,\mathrm{fm}$ lattice with $L = 1.3\,\mathrm{fm}$ and $m_\pi = 350\,\mathrm{MeV}$.
ALPHA 12	[29]	2	see ALPHA 04	2	Determination of $\Lambda_{\overline{\rm MS}}/f_K$ using ALPHA 04
Sternbeck 12	[218]	2	0.17-0.23	3	$\alpha_{\rm T}$ for $(r_0p)^2 = 200 - 2000$. Fit to PT without condensate. Deviation at higher energy is observed.
ETM 11C	[244]	2	0.26-0.96	3	$\alpha_{\rm qq}(1/r)$ as computed by us from $\Lambda_{\overline{\rm MS}} = 315\,\mathrm{MeV}$. Fit of $V(r)$ to PT with renormalon subtraction and resummation reproduces the static potential for $r/r_0 = 0.2 - 0.6$ well. One fit-range, using $r/a = 2 - 4$ at the smallest lattice spacing corresponds to $\alpha_{\rm eff} = 0.26 - 0.40$. In the $\overline{\rm MS}$ scheme one has $\alpha_{\overline{\rm MS}}(1/r) = 0.24 - 0.63$ and for the restricted fit $\alpha_{\overline{\rm MS}}(1/r) = 0.24-0.36$. Central values taken from $a = 0.042\,\mathrm{fm}$ lattice with $L = 1.3\,\mathrm{fm}$ and $m_\pi = 350\,\mathrm{MeV}$.
ETM 10F	[245]	2	0.24-0.45	3	α_T for momentum up to 2.6 - 5.6 GeV. Fitted to PT with gluon condensate correction term.
Sternbeck 10	[220]	2	0.19-0.38	3	$\alpha_{\rm T}$ for $1 \le (ap)^2 \le 10$. Fitted with $n_l = 3$ loop formula.
JLQCD 08	[246]	2	0.25-0.30	1	$\alpha_{\overline{\rm MS}}(Q)$ for $0.65 < (aQ)^2 < 1.32$. Fit with the perturbative formula with power corrections.
QCDSF-UKQCD 05	[224]	2	0.18-0.20	2	$\alpha_{\overline{\rm MS}}(1.4/a)$ computed from the boosted coupling.
ALPHA 04	[247]	2	0.078-0.44	2	$\alpha_{\rm SF}(1/L)$ step scaling, agreement with $n_l = 2$ looprunning for $\alpha_s < 0.2$
ALPHA 01	[248]	2	0.078-0.44	2	$\alpha_{\rm SF}(1/L)$ step scaling, agreement with $n_l = 2$ loop running for $\alpha_s < 0.2$
Boucaud 01B	[249]	2	0.25-0.5	3	$\alpha_{\widetilde{\rm MOM}}$ for momentum up to 7 GeV. Fitted with $n_l = 3$ loop formula with and without power correction, leading to different results for $\Lambda^{(2)}_{\overline{\rm MS}}$. Extrapolation of $\alpha_s(1.3\ \mathrm{GeV})$ in N_f from $N_f = 0, 2$ to $N_f = 3$ is made.
SESAM 99	[232]	2	0.17	1	The boosted coupling $\alpha_{\rm P}(3.41/a)$.
Wingate 95	[237]	2	0.18	1	$\alpha_{\rm V}(3.41/a)$ computed from the boosted coupling.
Aoki 94	[250]	2	0.14	1	$\alpha_{\overline{\rm MS}}(\pi/a)$ computed from the boosted coupling.
Davies 94	[238]	2	0.18	1	$\alpha_{\rm V}(3.41/a)$ computed from $\ln W_{11}$.

Table 160: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 2$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Bazavov 14	[251]	2+1	0.19-0.41	3	Update of Bazavov 12 including finer lattices down to $a = 0.041\,\mathrm{fm}$. Fit range $r/r_1 = 0.12 - 0.50$ ($r/r_0 = 0.08 - 0.33$). Perturbative expansion of the force $F(r)$ integrated to determine potential.
Bazavov 12	[252]	2+1	0.23-0.57	3	$\alpha_{\rm qq}$ computed by us from $\Lambda_{\overline{\rm MS}} r_0 = 0.70$. Fit of $V(r)$ to PT with renormalon subtraction and resummation reproduces the static potential for $r/r_0 = 0.135 - 0.5$ well.
Sternbeck 12	[218]	2+1	0.19-0.25	3	$\alpha_{\rm T}$ for $(pr_0)^2 = 200 - 2000$. Comparison with 4-loop formula.
JLQCD 10	[253]	2+1	0.29-0.35	2	$\alpha_{\overline{\rm MS}}(Q)$ for $0.4 < (aQ)^2 < 1.0$. Fit with the perturbative formula with power corrections.
HPQCD 10	[18]	2+1		2	Uses method of section 9.6. Update of r_1 and r_1/a in HPQCD 08A.
HPQCD 10	[18]	2+1	0.12-0.42	2	Uses method of section 9.7. $\alpha_{\rm eff}$ from R_4 and R_6/R_8. Fit of R_n, $n = 4 \ldots 10$ to PT including $(am)^{2i}$ terms with $i \leq 10$; coefficients constrained by priors.
PACS-CS 09A	[254]	2+1	0.08-0.27	2	$\alpha_{\rm SF}(1/L)$ step scaling, agreement with 3-loop running for $\alpha_s \leq 0.27$
HPQCD 08B	[49]	2+1	0.38	2	Fit of the ratios to PT at the charm mass including $(am)^{2i}$ terms with $i \leq 2 \ldots 4$; coefficients constrained by priors.
HPQCD 08A	[255]	2+1	0.15-0.4	2	$\alpha_{\rm V}(q^*)$ for a variety of short-distance quantities, using same method as in HPQCD 05A.
Maltman 08	[256]	2+1		2	Re-analysis of HPQCD 05A for a restricted set of short-distance quantities with similar results.
HPQCD 05A	[257]	2+1	0.2-0.4	2	$\alpha_{\rm V}(q^*)$ for a variety of short-distance quantities.

Table 161: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 3$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
HPQCD 14A	[1]	2+1+1	0.11-0.33	2	Range given for $\alpha_{\rm eff}$ from $\tilde{R}_4$. Fit of ratios R_n $n = 4 \dots 10$ to perturbation theory including $(am)^{2i}$ terms with $i \leq 10-20$ and higher-order perturbative terms; coefficients constrained by priors.
ETM 13D	[258]	2+1+1	0.26-0.7	3	$\alpha_{\rm T}(p)$ for $p = 1.6-6.5$ GeV. Update of [259] with improved power law determination.
ETM 12C	[259]	2+1+1	0.24-0.38	3	$\alpha_{\rm T}(p)$ for $p = 1.7-6.8$ GeV. Fit to PT with gluon condensate correction or higher power.
ALPHA 10A	[260]	4	0.07-0.28	2	$\alpha_{\rm SF}(1/L)$. Comparison to PT with 2-, 3-loop β-function.
ETM 11D	[261]	2+1+1	0.24-0.4	3	$\alpha_{\rm T}(p)$ for $p = 3.8-7.1$ GeV with H(4)-procedure. Fit to PT with gluon condensate correction.
Perez 10	[262]	4	0.06-0.28	2	$\alpha_{\rm SF}(1/L)$. Comparison with 1-, 2-, 3-loop β-function.

Table 162: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 4$.

B.7.2 Continuum limit

Collaboration	Ref.	N_f	$a\,\mu$	Description
FlowQCD 15	[217]	0	9 lattice spacings with $a =$ 0.06 - 0.02 fm.	$w_{0.4}/a$, together with $r_0 = 0.5$ fm and conversion factor $r_0/w_{0.4} = 2.587(45)$.
Sternbeck 12	[218]	0	4 lattice spacings $a \leq 0.1$ fm	At $\alpha_s = 0.18$, $ap = 2.7$, 1.5 for $\beta = 6.0, 6.4$.
Brambilla 10	[221]	0	At least 3 lattice spacings with $0.2 \leq 2a/r \leq 1.1$	Extrapolation of potential differences $V(r) - V(0.51r_0)$ linear in a^2 performed in [263] with several lattice spacings.
Ilgenfritz 10	[219]	0	a = 0.136, 0.093, 0.068, 0.051 fm (β = 5.8, 6.0, 6.2, 6.4), while no value of a is given for $\beta = 9.0$	At $\alpha_s = 0.3$, ap = 2.0, 1.4, 1.0, 0.8 (β = 5.8, 6.0, 6.2, 6.4). For $\beta = 9.0$ at $ap = 1.4$, $\alpha_s = 0.082$.
Sternbeck 10	[220]	0	8 lattice spacings $a = 0.004$ - 0.087 fm ($r_0 = 0.467$ fm)	$\sqrt{3} < ap < \sqrt{12}$.
Boucaud 08	[222]	0	$a = 0.1, 0.07, 0.05$ fm	At $\alpha_s = 0.3$ the data have $ap = 2.6, 1.9, 1.5$.
QCDSF/UKQCD 05	[224]	0	7 lattice spacings with $a =$ 0.10 - 0.028 fm.	r_0/a, together with $r_0 = 0.467$ fm.
Boucaud 05	[223]	0	$a = 0.1, 0.07, 0.05$ fm	At $\alpha_s \leq 0.3$ $ap = 1.9, 1.4, 1.0$.
CP-PACS 04	[225]	0	4 spacings, $a/L = 1/12 - 1/4$.	Iwasaki and Lüscher Weisz tree-level improved bulk actions; boundary improvement at tree-level, 1-loop and with two different choices of implementation.
Soto 01	[227]	0	$a = 0.07, 0.05, 0.03$ fm	At $\alpha_s \leq 0.3$, the data have $ap = 1.4, 1.0, 0.6$.
Boucaud 01A	[226]	0	$a = 0.1, 0.07, 0.05, 0.03$ fm	At $\alpha_s \leq 0.3$ $ap = 1.9, 1.4, 1.0, 0.6$.
Boucaud 00A	[228]	0	$a = 0.1, 0.07, 0.05, 0.03$ fm	At $\alpha_s \leq 0.3$ $ap = 1.9, 1.4, 1.0, 0.6$.
Boucaud 00B	[229]	0	$a = 0.1, 0.07, 0.05, 0.03$ fm	At $\alpha_s \leq 0.3$ $ap = 1.9, 1.4, 1.0, 0.6$.

Table 163: Continuum limit for α_s determinations with $N_f = 0$.

Collaboration	Ref.	N_f	$a\,\mu$	Description
SESAM 99	[232]	0	1 lattice spacing with $a = 0.086\,\mathrm{fm}$	Υ spectrum splitting.
Becirevic 99A	[230]	0	$a = 0.07, 0.05\,\mathrm{fm}$	At $\alpha_s \le 0.3$ $ap = 1.4, 1.0$.
Becirevic 99B	[231]	0	$a = 0.1, 0.07, 0.03\,\mathrm{fm}$	Only $a = 0.03$fm used to extract α_s. At $\alpha_s \le 0.3$, $ap = 0.6 - 1.5$.
ALPHA 98	[233]	0	4 to 6 spacings, $a/L = 1/12 - 1/5$ in step-scaling functions (SSF)	1-loop $\mathcal{O}(a)$ boundary improvement, linear extrapolation in a/L. $a/L = 1/8 - 1/5$ for $\alpha_s \le 0.11$ SSF, $a/L = 1/12 - 1/5$ for $0.12 \le \alpha_s \le 0.20$ SSF. L_{max}/r_0 from [264], where several lattice spacings were used.
Boucaud 98A	[234]	0	$a = 0.1, 0.07, 0.05\,\mathrm{fm}$	At $\alpha_s \le 0.3$, $ap = 1.9, 1.4, 1.0$.
Boucaud 98B	[235]	0	$a = 0.1, 0.07, 0.05\,\mathrm{fm}$	At $\alpha_s \le 0.3$, $ap = 1.9, 1.4, 1.0$.
Alles 96	[236]	0	$a \le 0.1\,\mathrm{fm}$	At $\alpha_s = 0.35$, $ap = 1.5$.
Wingate 95	[237]	0	1 lattice spacing with $a = 0.11\,\mathrm{fm}$	Charmonium $1S$-$1P$ splitting.
Davies 94	[238]	0	1 lattice spacing with $a = 0.077\,\mathrm{fm}$	Υ spectrum splitting.
Lüscher 93	[239]	0	4 or 5 lattice spacings, $a/L = 1/12 - 1/5$ in step-scaling functions	1-loop $\mathcal{O}(a)$ boundary improvement, linear extrapolation in a/L. $a/L = 1/8 - 1/5$ for $\alpha_s \le 0.11$ SSF, $a/L = 1/10 - 1/5$ for $0.11 \le \alpha_s \le 0.22$ SSF, $a/L = 1/12 - 1/6$ for $0.22 \le \alpha_s \le 0.28$ SSF, $a/L = 1/8.5 - 1/4.5$ for continuum extrapolation of $L_{\mathrm{max}}/\sqrt{K}$.
UKQCD 92	[240]	0	One lattice spacing with $0.44 \le 2a/r \le 1.6$	No continuum limit.
Bali 92	[241]	0	One lattice spacing with $0.4 \le 2a/r \le 1.6$	No continuum limit.
El-Khadra 92	[242]	0	3 lattice spacings with $a = 0.17,\ 0.11,\ 0.08\,\mathrm{fm}$	Charmonium $1S$-$1P$ splitting.

Table 163: Continuum limit for α_s determinations with $N_f = 0$ continued.

Collab.	Ref.	N_f	$a\,\mu$	Description
Karbstein 14	[243]	2	0.32 – 1.19 0.63 – 1.19	at $p = 1.5\,\mathrm{GeV}$ at $p = 3\,\mathrm{GeV}$, roughly coincides with $\alpha_s = 0.3$.
ALPHA 12	[29]	2	$a = 0.049, 0.066, 0.076$fm from f_K	2-loop $\mathcal{O}(a)$ boundary improvement, linear extrapolation of $L_{\max} f_K$ in a^2.
Sternbeck 12	[218]	2	$a = 0.073, 0.07, 0.06\,\mathrm{fm}$	At $\alpha_s = 0.23$, $ap = 2.1, 2.0, 1.7$.
ETM 11C	[244]	2	$0.30 \leq 2a/r \leq 1.0$ $0.67 \leq 2a/r \leq 1.26$ when $\alpha_s = 0.3$	Four lattice spacings; continuum limit studied with a particular range in r; central result from the smallest lattice spacing, $a = 0.042$fm.
ETM 10F	[245]	2	$a = 0.05, 0.07, 0.08$ fm. Different lattice spacings are patched together.	At $\alpha_s = 0.3$, $ap = 1.6, 1.3, 1.1$.
Sternbeck 10	[220]	2	$a = 0.068, 0.076, 0.082$ fm	At $\alpha_s \leq 0.3$, $ap \geq 1.7$.
JLQCD 08	[246]	2	$a = 0.12$ fm from $r_0 = 0.49\,\mathrm{fm}$	Single lattice spacing, $0.64 < (aQ)^2 < 1.32$. At $\alpha_s = 0.3$, $ap = 0.81$.
QCDSF-UKQCD 05	[224]	2	4 lattice spacings with $a =$ 0.10 - 0.066 fm	r_0, together with $r_0 = 0.467\,\mathrm{fm}$.
ALPHA 04	[247]	2	$a/L = 1/8, 1/6, 1/5, 1/4$	1-loop (at weak coupling) and 2-loop $\mathcal{O}(a)$ boundary improvement, linear extrapolation of SSF in $(a/L)^2$
ALPHA 01A	[248]	2	$a/L = 1/6, 1/5, 1/4$	1-loop (at weak coupling) and 2-loop $\mathcal{O}(a)$ boundary improvement, weighted average of SSF with $a/L = 1/5, 1/6$.
Boucaud 01B	[249]	2	$a = 0.05, 0.07, 0.09\,\mathrm{fm}$. Data at different lattice spacings are patched together	At $\alpha_s = 0.3$, $ap = 1.6, 1.3, 0.9$; plain Wilson action with $\mathcal{O}(a)$ errors.
SESAM 99	[232]	2	1 lattice spacing with $a = 0.079\,\mathrm{fm}$	Υ spectrum splitting.
Wingate 95	[237]	2	1 lattice spacing with $a = 0.11\,\mathrm{fm}$	Charmonium $1S$-$1P$ splitting.
Aoki 94	[250]	2	1 lattice spacing with $a = 0.10\,\mathrm{fm}$	Charmonium $1P - 1S$ splitting
Davies 94	[238]	2	1 lattice spacing with $a = 0.08\,\mathrm{fm}$	Υ spectrum splitting.

Table 164: Continuum limit for α_s determinations with $N_f = 2$.

Collab.	Ref.	N_f	$a\,\mu$	Description
Bazavov 14	[251]	2+1	$2a/r = 0.52 - 3.2$	5 lattice spacings; 3 used for determination. At $\alpha_{\rm eff} = 0.3$, then $0.86 < a\mu = 2a/r < 1.3$. $m_\pi L = 2.4, 2.6, 2.2$ at smallest three lattice spacings of $a = 0.060, 0.049, 0.041$fm respectively [265]; adequate coverage of topological sectors is not clear.
Bazavov 12	[252]	2+1	$2a/r = 0.6 - 2.0$	7 lattice spacings; 4 lattice spacings with $1.14 \le 2a/r \le 1.5$ when $\alpha_s(1/r) = 0.3$. $2a/r = 2$ when $\alpha_s(1/r) = 0.23$ (on the finest lattice).
Sternbeck 12	[218]	2+1	$a = 0.07$ fm	At $\alpha_s = 0.23$, $ap = 2.1$.
HPQCD 10	[18]	2+1	$a\mu = 2a\bar{m}_h = 0.61 - 1.75$	5 lattice spacings; 3 lattice spacings with $1.0 \le a\mu \le 1.5$ when $\alpha_{R_4}(\mu) \le 0.3$; 3 lattice spacings with $1.0 \le a\mu \le 1.5$ when $\alpha_{R_6/R_8}(\mu) \le 0.33$.
JLQCD 10	[253]	2+1	$a = 0.11$ fm from $r_0 = 0.49$ fm	Single lattice spacing, $0.4 < (aQ)^2 < 1.0$ for the momentum fit range. At $\alpha_s = 0.3$, $ap = 0.89$.
HPQCD 10	[18]	2+1		Update of r_1 and r_1/a in HPQCD 08A.
PACS-CS 09A	[254]	2+1	$a/L = 1/8, 1/6, 1/4$	Tree-level $\mathcal{O}(a)$ boundary improvement, which has been seen to behave better than 1-loop in simulations [225]; weighted average of $a/L = 1/8, 1/6$ for step-scaling function which agrees with a linear extrapolation in a/L of all data points of the SSF. Linear extrapolation in a/L of $L_{\rm max} m_\rho$ with $a/L_{\rm max} = 1/8, 1/6, 1/4$.
HPQCD 08B	[49]	2+1	$a\mu = 2a\bar{m}_h = 0.8,\ 1.2,\ 1.7,\ 2.1$	4 lattice spacings with heavy quark mass approximately the charm mass, where $\alpha_{R_4}(\mu) = 0.38$.
HPQCD 08A	[255]	2+1	6 lattice spacings with $a =$ 0.18 - 0.045 fm	r_1 using Υ spectrum splitting.
Maltman 08	[256]	2+1	5 lattice spacings with $a =$ 0.18 - 0.06 fm	Re-analysis of HPQCD 05A with additional lattice spacings $a = 0.06$, 0.15 fm.
HPQCD 05A	[257]	2+1	3 lattice spacings with $a =$ 0.18 - 0.09 fm	r_1 using Υ spectrum splitting.

Table 165: Continuum limit for α_s determinations with $N_f = 3$.

Collab.	Ref.	N_f	$a\,\mu$	Description
HPQCD 14A	[1]	2+1+1	$a\mu = 2a\bar{m}_h = 0.78 - 2.09$	4 lattice spacings; 2 lattice spacings with $a\mu \le 1.5$ and one more lattice spacing with $a\mu \lesssim 1.6$ when $\alpha_{R_4}(\mu) \le 0.3$.
ETM 13D	[258]	2+1+1	$a = 0.060, 0.068$ fm from f_π	For $\alpha_s \le 0.3$, $ap = 1.5, 1.7$. Update of [259].
ETM 12C	[259]	2+1+1	$a = 0.061, 0.078$ fm from f_π	Global fit with $(ap)^2$ discretization effects. For $\alpha_s \le 0.3$, $ap = 1.5, 2.2$.
ETM 11D	[261]	2+1+1	$a = 0.061, 0, 078\,\mathrm{fm}$	For $\alpha_s \le 0.3$, $ap = 1.5, 2.0$.
ALPHA 10A	[260]	4	$a/L = 1/4, 1/6, 1/8$	Constant or global linear fit in $(a/L)^2$.
Perez 10	[262]	4	$a/L = 1/4, 1/6, 1/8$	Linear extrapolation in $(a/L)^2$. 1-loop improvement at the boundary.

Table 166: Continuum limit for α_s determinations with $N_f = 4$.

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