

## 10 Nucleon matrix elements (NME)

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A large number of experiments testing the Standard Model (SM) and searching for physics Beyond the Standard Model (BSM) involve either free nucleons (proton and neutron beams) or the scattering of electrons, muons, neutrinos and dark matter off nuclear targets. Necessary ingredients in the analysis of the experimental results are the matrix elements of various probes (fundamental currents or operators in a low energy effective theory) between nucleon or nuclear states. The goal of lattice-QCD calculations in this context is to provide high precision predictions of these matrix elements, the simplest of which give the nucleon charges and form factors. Determinations of the charges are the most mature and in this review we summarize the results for twelve quantities, the isovector and flavour diagonal axial vector, scalar and tensor charges. Other quantities that are not being reviewed but for which significant progress has been made in the last five years are the nucleon axial vector and electromagnetic form factors [1–15] and parton distribution functions [16–20]. The more challenging calculations of nuclear matrix elements, that are needed, for example, to calculate the cross-sections of neutrinos or dark matter scattering off nuclear targets, are proceeding along three paths. First is direct evaluation of matrix elements calculated with initial and final states consisting of multiple nucleons [21, 22]. Second, convoluting nucleon matrix elements with nuclear effects [23], and third, determining two and higher body terms in the nuclear potential via the direct or the HAL QCD methods [24, 25]. We expect future FLAG reviews to include results on these quantities once a sufficient level of control over all the systematics is reached.

### 10.1 Isovector and flavour diagonal charges of the nucleon

The simplest nucleon matrix elements are composed of local quark bilinear operators,  $\bar{q}_i \Gamma_\alpha q_j$ , where  $\Gamma_\alpha$  can be any of the sixteen Dirac matrices. In this report, we consider two types of flavour structures: (a) when  $i = u$  and  $j = d$ . These  $\bar{u} \Gamma_\alpha d$  operators arise in  $W^\pm$  mediated weak interactions such as in neutron or pion decay. We restrict the discussion to the matrix elements of the axial vector ( $A$ ), scalar ( $S$ ) and tensor ( $T$ ) currents, which give the isovector charges,  $g_{A,S,T}^{u-d}$ .<sup>1</sup> (b) When  $i = j$  for  $j \in \{u, d, s\}$ , there is no change of flavour, e.g., in processes mediated via the electromagnetic or weak neutral interaction or dark matter. These  $\gamma$  or  $Z^0$  or possible dark matter mediated processes couple to all flavours with their corresponding charges. Since these probes interact with nucleons within nuclear targets, one has to include the effects of QCD (to go from the couplings defined at the quark and gluon level to those for nucleons) and nuclear forces in order to make contact with experiments. The isovector and flavour diagonal charges, given by the matrix elements of the corresponding operators calculated between nucleon states, are these nucleon level couplings. Here we review results for the light and strange flavours,  $g_{A,S,T}^u$ ,  $g_{A,S,T}^d$ , and  $g_{A,S,T}^s$  and the isovector charges  $g_{A,S,T}^{u-d}$ .

The isovector and flavour diagonal operators also arise in BSM theories due to the exchange of novel force carriers or as effective interactions due to loop effects. The associated couplings are defined at the energy scale  $\Lambda_{\text{BSM}}$ , while lattice-QCD calculations of matrix elements are

<sup>1</sup>In the isospin symmetric limit  $\langle p | \bar{u} \Gamma d | n \rangle = \langle p | \bar{u} \Gamma u - \bar{d} \Gamma d | p \rangle = \langle n | \bar{d} \Gamma d - \bar{u} \Gamma u | n \rangle$  for nucleon and proton states  $|p\rangle$  and  $|n\rangle$ , respectively. The latter two (equivalent) isovector matrix elements are computed on the lattice.

carried out at a hadronic scale,  $\mu$ , of a few GeV. The tool for connecting the couplings at the two scales is the renormalization group. Since the operators of interest are composed of quark fields (and more generally also of gluon fields), the predominant change in the corresponding couplings under a scale transformation is due to QCD. To define the operators and their couplings at the hadronic scale  $\mu$ , one constructs renormalized operators, whose matrix elements are finite in the continuum limit. This requires calculating both multiplicative renormalization factors, including the anomalous dimensions and finite terms, and the mixing with other operators. We discuss the details of the renormalization factors needed for each of the six operators reviewed in this report in Sec. 10.1.3.

Once renormalized operators are defined, the matrix elements of interest are extracted using expectation values of two-point and three-point correlation functions illustrated in Fig. 42, where the latter can have both quark-line connected and disconnected contributions. In order to isolate the ground-state matrix element, these correlation functions are analyzed using their spectral decomposition. The current practice is to fit the  $n$ -point correlation functions (or ratios involving three- and two-point functions) including contributions from one or two excited states. In some cases, such as axial and vector operators, Ward identities provide relations between correlation functions, or ground state matrix elements, or facilitate the calculation of renormalization constants. It is important to ensure that all such Ward identities are satisfied in lattice calculations, especially as in the case of axial form factors where they provide checks of whether excited state contamination has been removed in obtaining matrix elements within ground state nucleons [12, 26, 27].

The ideal situation occurs if the time separation  $\tau$  between the nucleon source and sink positions, and the distance of the operator insertion time from the source and the sink,  $t$  and  $\tau - t$ , respectively, are large enough such that the contribution of all excited states is negligible. In the limit of large  $\tau$ , the ratio of noise to signal in the nucleon two- and three-point correlation functions grows exponentially as  $e^{(M_N - \frac{3}{2}M_\pi)\tau}$  [28, 29], where  $M_N$  and  $M_\pi$  are the masses of the nucleon and the pion, respectively. Therefore, in particular at small pion masses, maintaining reasonable errors for large  $\tau$  is challenging, with current calculations limited to  $\tau \lesssim 1.5$  fm. In addition, the mass gap between the ground and excited (including multi-particle) states is smaller than in the meson sector and at these separations, excited-state effects can be significant. The approach commonly taken is to first obtain results with high statistics at multiple values of  $\tau$ , using the methods described in Sec. 10.1.1. Then, as mentioned above, excited-state contamination is removed by fitting the data using a fit form involving one or two excited states. The different strategies that have been employed to minimize excited-state contamination are discussed in Sec. 10.1.2.

Usually, the quark-connected part of the three-point function (corresponding to the plot in the centre of Fig. 42) is computed via the so-called “sequential propagator method”, which uses the product of two quark propagators between the positions of the initial and the final nucleons as a source term for another inversion of the lattice Dirac operator. This implies that the position of the sink timeslice is fixed at some chosen value. Varying the value of the source-sink separation  $\tau$  then requires the calculation of another sequential propagator.

The evaluation of quark-disconnected contributions is computationally more challenging as the disconnected loop (which contains the operator insertion, as illustrated in Fig. 42 right) is needed at all points on a particular timeslice or, in general, over the whole lattice. The quark loop is computed stochastically and then correlated with the nucleon two-point function before averaging this three-point function over the ensemble of gauge configurations. The associated statistical error, therefore, is a combination of that due to the stochastic eval-

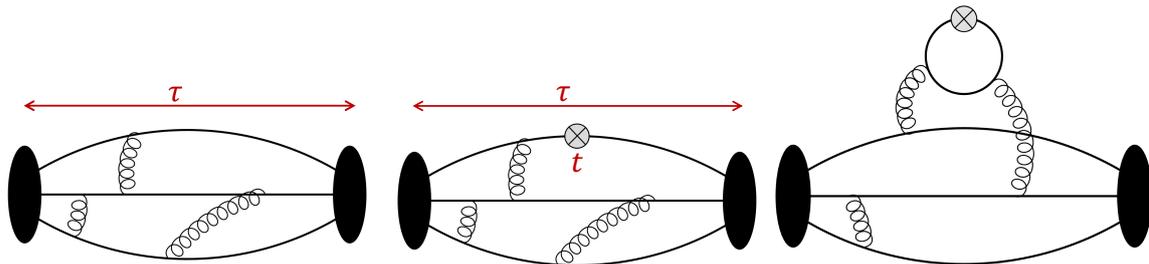


Figure 42: The two- and three-point correlation functions (illustrated by Feynman diagrams) that need to be calculated to extract the ground state nucleon matrix elements. (Left) the nucleon two-point function. (Middle) the connected three-point function with source-sink separation  $\tau$  and operator insertion time slice  $t$ . (Right) the disconnected three-point function with operator insertion at  $t$ .

uation (on each configuration) and that from the gauge average. The number of stochastic sources employed on each configuration is, typically, optimized to reduce the overall error for a given computational cost. The statistical errors of the connected contributions, in contrast, usually come only from the ensemble average since they are often evaluated exactly on each configuration, for a small number of source positions. If these positions are well-separated in space and time, then each measurement is statistically independent. The methodology applied for these calculations and the variance reduction techniques are summarized in Sec. 10.1.1. By construction, arbitrary values of  $\tau$  across the entire temporal extent of the lattice can be realized when computing the quark-disconnected contribution, since the source-sink separation is determined by the part of the diagram that corresponds to the two-point nucleon correlator. However, in practice statistical fluctuations of both the connected and disconnected contributions increase sharply, so that the signal is lost in the statistical noise for  $\tau \gtrsim 1.5$  fm.

The lattice calculation is performed for a given number of quark flavours and at a number of values of the lattice spacing  $a$ , the pion mass  $M_\pi$ , and the lattice size, represented by  $M_\pi L$ . The results need to be extrapolated to the physical point defined by  $a = 0$ ,  $M_\pi = 135$  MeV and  $M_\pi L \rightarrow \infty$ . This is done by fitting the data simultaneously in these three variables using a theoretically motivated ansatz. The ansätze used and the fitting strategy are described in Sec. 10.1.4.

The procedure for rating the various calculations and the criteria specific to this chapter are discussed in Sec. 10.2, which also includes a brief description of how the final averages are constructed. The physics motivation for computing the isovector charges,  $g_{A,S,T}^{u-d}$ , and the review of the lattice results are presented in Sec. 10.3. This is followed by a discussion of the relevance of the flavour diagonal charges,  $g_{A,S,T}^{u,d,s}$ , and a presentation of the lattice results in Sec. 10.4.

### 10.1.1 Technical aspects of the calculations of nucleon matrix elements

The calculation of  $n$ -point functions needed to extract nucleon matrix elements requires making four essential choices. The first involves choosing between the suite of background gauge field ensembles one has access to. The range of lattice parameters should be large enough to facilitate the extrapolation to the continuum and infinite-volume limits, and, ideally, the evaluation at the physical pion mass taken to be  $M_\pi = 135$  MeV. Such ensembles have been

generated with a variety of discretization schemes for the gauge and fermion actions that have different levels of improvement and preservation of continuum symmetries. The actions employed at present include (i) Wilson gauge with nonperturbatively improved Sheikholeslami-Wohlert fermions (nonperturbatively improved clover fermions) [30–36], (ii) Iwasaki gauge with nonperturbatively improved clover fermions [9, 37], (iii) Iwasaki gauge with twisted-mass fermions with a clover term [38–42], (iv) tadpole Symanzik improved gauge with highly improved staggered quarks (HISQ) [43–51], (v) Iwasaki gauge with domain-wall fermions (DW) [26, 52–57] and (vi) Iwasaki gauge with overlap fermions [58–60]. For details of the lattice actions, see the glossary in the Appendix A.1 of FLAG 19 [61].

The second choice is of the valence quark action. Here there are two choices, to maintain a unitary formulation by choosing exactly the same action as is used in the generation of gauge configurations or to choose a different action and tune the quark masses to match the pseudoscalar meson spectrum in the two theories. Such mixed action formulations are nonunitary but are expected to have the same continuum limit as QCD. The reason for choosing a mixed action approach is expediency. For example, the generation of 2+1+1 flavour HISQ and 2+1 flavour DW ensembles with physical quark masses has been possible even at the coarse lattice spacing of  $a = 0.15$  fm and there are indications that cut-off effects are reasonably small. These ensembles have been analyzed using clover-improved Wilson fermions, DW and overlap fermions since the construction of baryon correlation functions with definite spin and parity is much simpler compared to staggered fermions.

The third choice is the combination of the algorithm for inverting the Dirac matrix and variance reduction techniques. Efficient inversion and variance reduction techniques are needed for the calculation of nucleon correlation functions with high precision because the signal-to-noise ratio degrades exponentially as  $e^{(\frac{3}{2}M_\pi - M_N)\tau}$  with the source-sink separation  $\tau$ . Thus, the number of measurements needed for high precision is much larger than in the meson sector. Commonly used inversion algorithms include the multigrid [62] and the deflation-accelerated Krylov solvers [63], which can handle linear systems with large condition numbers very efficiently, thereby enabling calculations of correlation functions at the physical pion mass.

The sampling of the path integral is limited by the number  $N_{\text{conf}}$  of gauge configurations generated. One requires sufficiently large  $N_{\text{conf}}$  such that the phase space (for example, different topological sectors) has been adequately sampled and all the correlation functions satisfy the expected lattice symmetries such as  $C$ ,  $P$ ,  $T$ , momentum and translation invariance. Thus, one needs gauge field generation algorithms that give decorrelated large volume configurations cost-effectively. On such large lattices, to reduce errors one can exploit the fact that the volume is large enough to allow multiple measurements of nucleon correlation functions that are essentially statistically independent. Two other common variance reduction techniques that reduce the cost of multiple measurements on each configuration are: the truncated solver with bias correction method [64] and deflation of the Dirac matrix for the low lying modes followed by sloppy solution with bias correction for the residual matrix consisting predominately of the high frequency modes [64, 65].

A number of other variance reduction methods are also being used and developed. These include deflation with hierarchical probing for disconnected diagrams [66, 67], the coherent source sequential propagator method [68, 69], low-mode averaging [70, 71], the hopping-parameter expansion [72, 73] and partitioning [74] (also known as dilution [75]).

The final choice is of the interpolating operator used to create and annihilate the nucleon state, and of the operator used to calculate the matrix element. Along with the choice of

the interpolating operator (or operators if a variational method is used) one also chooses a “smearing” of the source used to construct the quark propagator. By tuning the width of the smearing, one can optimize the spatial extent of the nucleon interpolating operator to reduce the overlap with the excited states. Two common smearing algorithms are Gaussian (Wuppertal) [76] and Jacobi [77] smearing.

Having made all the above choices, for which a reasonable recipe exists, one calculates a statistical sample of correlation functions from which the desired ground state nucleon matrix element is extracted. Excited states, unfortunately, contribute significantly to nucleon correlation functions in present studies. To remove their contributions, calculations are performed with multiple source-sink separations  $\tau$  and fits are made to the correlation functions using their spectral decomposition as discussed in the next section.

### 10.1.2 Controlling excited-state contamination

Nucleon matrix elements are determined from a combination of two- and three-point correlation functions. To be more specific, let  $B^\alpha(\vec{x}, t)$  denote an interpolating operator for the nucleon. Placing the initial state at time slice  $t = 0$ , the two-point correlation function of a nucleon with momentum  $\vec{p}$  reads

$$C_2(\vec{p}; \tau) = \sum_{\vec{x}, \vec{y}} e^{i\vec{p} \cdot (\vec{x} - \vec{y})} \mathbb{P}_{\beta\alpha} \left\langle B^\alpha(\vec{x}, \tau) \bar{B}^\beta(\vec{y}, 0) \right\rangle, \quad (415)$$

where the projector  $\mathbb{P}$  selects the polarization, and  $\alpha, \beta$  denote Dirac indices. The three-point function of two nucleons and a quark bilinear operator  $O_\Gamma$  is defined as

$$C_3^\Gamma(\vec{q}; t, \tau) = \sum_{\vec{x}, \vec{y}, \vec{z}} e^{i\vec{p}' \cdot (\vec{x} - \vec{z})} e^{-i\vec{p} \cdot (\vec{y} - \vec{z})} \mathbb{P}_{\beta\alpha} \left\langle B^\alpha(\vec{x}, \tau) O_\Gamma(\vec{z}, t) \bar{B}^\beta(\vec{y}, 0) \right\rangle, \quad (416)$$

where  $\vec{p}$ ,  $\vec{p}'$  denote the momenta of the nucleons at the source and sink, respectively, and  $\vec{q} \equiv \vec{p}' - \vec{p}$  is the momentum transfer. The bilinear operator is inserted at time slice  $t$ , and  $\tau$  denotes the source-sink separation. Both  $C_2$  and  $C_3^\Gamma$  can be expressed in terms of the nonperturbative quark propagators,  $D^{-1}(y, x)$ , where  $D$  denotes the lattice Dirac operator.

The framework for the analysis of excited-state contamination is based on spectral decomposition. After inserting complete sets of eigenstates of the transfer matrix, the expressions for the correlators  $C_2$  and  $C_3^\Gamma$  read

$$C_2(\vec{p}; \tau) = \frac{1}{L^3} \sum_n \mathbb{P}_{\beta\alpha} \langle \Omega | B^\alpha | n \rangle \langle n | \bar{B}^\beta | \Omega \rangle e^{-E_n \tau}, \quad (417)$$

$$C_3^\Gamma(\vec{q}; t, \tau) = \frac{1}{L^3} \sum_{n, m} \mathbb{P}_{\beta\alpha} \langle \Omega | B^\alpha | n \rangle \langle n | O_\Gamma | m \rangle \langle m | \bar{B}^\beta | \Omega \rangle e^{-E_n(\tau-t)} e^{-E_m t}, \quad (418)$$

where  $|\Omega\rangle$  denotes the vacuum state, and  $E_n$  represents the energy of the  $n^{\text{th}}$  eigenstate  $|n\rangle$  in the nucleon channel. Here we restrict the discussion to vanishing momentum transfer, i.e., the forward limit  $\vec{q} = 0$ , and label the ground state by  $n = 0$ . The matrix element of interest  $g_\Gamma \equiv \langle 0 | O_\Gamma | 0 \rangle$  can, for instance, be obtained from the asymptotic behaviour of the ratio

$$R_\Gamma(t, \tau) \equiv \frac{C_3^\Gamma(\vec{q} = 0; t, \tau)}{C_2(\vec{p} = 0; \tau)} \xrightarrow{t, (\tau-t) \rightarrow \infty} g_\Gamma + \mathcal{O}(e^{-\Delta t}, e^{-\Delta(\tau-t)}, e^{-\Delta\tau}), \quad (419)$$

where  $\Delta \equiv E_1 - E_0$  denotes the energy gap between the ground state and the first excitation. We also assume that the bilinear operator  $O_\Gamma$  is appropriately renormalized (see Sec. 10.1.3).

Excited states with the same quantum numbers as the nucleon include resonances such as a Roper-like state with a mass of about 1.5 GeV, or multi-particle states consisting of a nucleon and one or more pions [78, 79]. The latter can provide significant contributions to the two- and three-point correlators in Eqs. (415) and (416) or their ratios (419) as the pion mass approaches its physical value. Ignoring the interactions between the individual hadrons, one can easily identify the lowest-lying multi-particle states: they include the  $N\pi\pi$  state with all three particles at rest at  $\sim 1.2$  GeV, as well as  $N\pi$  states with both hadrons having nonzero and opposite momentum. Depending on the spatial box size  $L$  in physical units (with the smallest nonzero momentum equal to  $2\pi/L$ ), there may be a dense spectrum of  $N\pi$  states before the first nucleon resonance is encountered. Corrections to nucleon correlation functions due to the pion continuum have been studied using chiral effective theory [78–81] and Lüscher’s finite-volume quantization condition [82].

The well-known noise problem of baryonic correlation functions implies that the long-distance regime,  $t, (\tau - t) \rightarrow \infty$ , where the correlators are dominated by the ground state, is difficult to reach. Current lattice calculations of baryonic three-point functions are typically confined to source-sink separations of  $\tau \lesssim 1.5$  fm, despite the availability of efficient noise reduction methods. In view of the dense excitation spectrum encountered in the nucleon channel, one has to demonstrate that the contributions from excited states are sufficiently suppressed to guarantee an unbiased determination of nucleon matrix elements. There are several strategies to address this problem:

- Multi-state fits to correlator ratios or individual two- and three-point functions;
- Three-point correlation functions summed over the operator insertion time  $t$ ;
- Increasing the projection of the interpolator  $B^\alpha$  onto the ground state.

The first of the above methods includes excited state contributions explicitly when fitting to the spectral decomposition of the correlation functions, Eqs. (417, 418) or, alternatively, their ratio (see Eq. (419)). In its simplest form, the resulting expression for  $R_\Gamma$  includes the contributions from the first excited state, i.e.,

$$R_\Gamma(t, \tau) = g_\Gamma + c_{01} e^{-\Delta t} + c_{10} e^{-\Delta(\tau-t)} + c_{11} e^{-\Delta\tau} + \dots, \quad (420)$$

where  $c_{01}, c_{10}, c_{11}$  and  $\Delta$  are treated as additional parameters when fitting  $R_\Gamma(t, \tau)$  simultaneously over intervals in the source-sink separation  $\tau$  and the operator insertion timeslice  $t$ . Multi-exponential fits become more difficult to stabilize for a growing number of excited states, since an increasing number of free parameters must be sufficiently constrained by the data. Therefore, a high level of statistical precision at several source-sink separations is required. One common way to address this issue is to introduce Bayesian constraints, as described in [83]. Alternatively, one may try to reduce the number of free parameters, for instance, by determining the energy gap  $\Delta$  from nucleon two-point function and/or using a common gap for several different nucleon matrix elements [84].

Ignoring the explicit contributions from excited states and fitting  $R_\Gamma(t, \tau)$  to a constant in  $t$  for fixed  $\tau$  amounts to applying what is called the “plateau method”. The name derives from the ideal situation that sufficiently large source-sink separations  $\tau$  can be realized, which

would cause  $R_\Gamma(t, \tau)$  to exhibit a plateau in  $t$  independent of  $\tau$ . The ability to control excited-state contamination is rather limited in this approach, since the only option is to check for consistency in the estimate of the plateau as  $\tau$  is varied. In view of the exponential degradation of the statistical signal for increasing  $\tau$ , such stability checks are difficult to perform reliably.

Summed operator insertions, originally proposed in Ref. [85], have also emerged as a widely used method to address the problem of excited-state contamination. One way to implement this method [86, 87] proceeds by summing  $R_\Gamma(t, \tau)$  over the insertion time  $t$ , resulting in the correlator ratio  $S_\Gamma(\tau)$ ,

$$S_\Gamma(\tau) \equiv \sum_{t=a}^{\tau-a} R_\Gamma(t, \tau). \quad (421)$$

The asymptotic behaviour of  $S_\Gamma(\tau)$ , including sub-leading terms, for large source-sink separations  $\tau$  can be easily derived from the spectral decomposition of the correlators and is given by [88]

$$S_\Gamma(\tau) \xrightarrow{\tau \gg 1/\Delta} K_\Gamma + (\tau - a) g_\Gamma + (\tau - a) e^{-\Delta\tau} d_\Gamma + e^{-\Delta\tau} f_\Gamma + \dots, \quad (422)$$

where  $K_\Gamma$  is a constant, and the coefficients  $d_\Gamma$  and  $f_\Gamma$  contain linear combinations of transition matrix elements involving the ground and first excited states. Thus, the matrix element of interest  $g_\Gamma$  is obtained from the linear slope of  $S_\Gamma(\tau)$  with respect to the source-sink separation  $\tau$ . While the leading corrections from excited states  $e^{-\Delta\tau}$  are smaller than those of the original ratio  $R_\Gamma(t, \tau)$  (see Eq. (419)), extracting the slope from a linear fit to  $S_\Gamma(\tau)$  typically results in relatively large statistical errors. In principle, one could include the contributions from excited states explicitly in the expression for  $S_\Gamma(\tau)$ . However, in practice it is often difficult to constrain an enlarged set of parameters reliably, in particular if one cannot afford to determine  $S_\Gamma(\tau)$  except for a handful of source-sink separations.

The original summed operator insertion technique described in Refs. [76, 85, 89, 90] avoids the explicit summation over the operator insertion time  $t$  at every fixed value of  $\tau$ . Instead, one replaces one of the quark propagators that appear in the representation of the two-point correlation function  $C_2(t)$  by a ‘‘sequential’’ propagator, according to

$$D^{-1}(y, x) \rightarrow D_\Gamma^{-1}(y, x) = \sum_z D^{-1}(y, z) \Gamma D^{-1}(z, x). \quad (423)$$

In this expression, the position  $z \equiv (\vec{z}, t)$  of the insertion of the quark bilinear operator is implicitly summed over, by inverting the lattice Dirac operator  $D$  on the source field  $\Gamma D^{-1}(z, x)$ . While this gives access to all source-sink separations  $0 \leq \tau \leq T$ , where  $T$  is the temporal extent of the lattice, the resulting correlator also contains contact terms, as well as contributions from  $\tau < t < T$  that must be controlled. This method<sup>2</sup> has been adopted recently by the CalLat collaboration in their calculation of the isovector axial charge [47, 51].

As in the case of explicitly summing over the operator insertion time, the matrix element of interest is determined from the slope of the summed correlator. For instance, in Ref. [51], the axial charge was determined from the summed three-point correlation function, by fitting to its asymptotic behaviour [91] including sub-leading terms.

In practice, one often uses several methods simultaneously, e.g., multi-state fits and the summation method based on Eq. (422), in order to check the robustness of the result. All of

<sup>2</sup>In Ref. [91] it is shown that the method can be linked to the Feynman-Hellmann theorem. A direct implementation of the Feynman-Hellmann theorem by means of a modification of the lattice action is discussed and applied in Refs. [92, 93].

the approaches for controlling excited-state contributions proceed by fitting data obtained in a finite interval in  $\tau$  to a function that describes the approach to the asymptotic behaviour derived from the spectral decomposition. Obviously, the accessible values of  $\tau$  must be large enough so that the model function provides a good representation of the data that enter such a fit. It is then reasonable to impose a lower threshold on  $\tau$  above which the fit model is deemed reliable. We will return to this issue when explaining our quality criteria in Sec. 10.2.

The third method for controlling excited-state contamination aims at optimizing the projection onto the ground state in the two-point and three-point correlation functions [34, 69, 94, 95]. The RQCD collaboration has chosen to optimize the parameters in the Gaussian smearing procedure, so that the overlap of the nucleon interpolating operator onto the ground state is maximized [34]. In this way it may be possible to use shorter source-sink separations without incurring a bias due to excited states.

The variational method, originally designed to provide detailed information on energy levels of the ground and excited states in a given channel [96–99], has also been adapted to the determination of hadron-to-hadron transition elements [88]. In the case of nucleon matrix elements, the authors of Ref. [94] have employed a basis of operators to construct interpolators that couple to individual eigenstates in the nucleon channel. The method has produced promising results when applied to calculations of the axial and other forward matrix elements at a fixed value of the pion mass [69, 94, 95, 100]. However, a more comprehensive study aimed at providing an estimate at the physical point has, until now, not been performed.

The investigation of excited-state effects is an active subfield in NME calculations, and many refinements and extensions have been implemented since the previous edition of the FLAG report. For instance, it has been shown that the previously observed failure of the axial and pseudoscalar form factors to satisfy the PCAC relation linking them could be avoided by including the enhanced contribution of  $N\pi$  excitations, either by including additional information on the nucleon excitation spectrum extracted from the three-point function of the axial current [27], or with guidance from chiral effective field theory analyses of nucleon three-point functions [12].

The variety of methods that are employed to address the problem of excited-state contamination (ESC) has greatly improved our understanding of and control over excited-state effects in NME calculations. However, there is still room for further improvement: For instance, dedicated calculations of the excitation spectrum using the variational method could replace the often rudimentary spectral information gained from multi-state fits to the two- and three-point functions used primarily for the determination of the matrix elements. In general, the development of methods to explicitly include multi-particle states, such as  $N\pi$  and  $N\pi\pi$  with appropriate momentum configurations, coupled with the determination of the associated (transition) matrix elements, is needed to significantly enhance the precision of a variety of nucleon matrix elements. Such approaches would, to some extent, eliminate the need to extend the source-sink separation  $\tau$  into a regime that is currently inaccessible due to the noise problem.

Since the ongoing efforts to study excited-state contamination are producing deeper insights, we have decided to follow a more cautious approach in the assessment of available NME calculations. This is reflected in a modification of the quality criterion for excited-state contamination that is described and discussed in Sec. 10.2.

### 10.1.3 Renormalization and Symanzik improvement of local currents

In this section we discuss the matching of the normalization of lattice operators to a continuum reference scheme such as  $\overline{\text{MS}}$ , and the application of Symanzik improvement to remove  $\mathcal{O}(a)$  contributions. The relevant operators for this review are the axial ( $A_\mu$ ), tensor ( $T_{\mu\nu}$ ) and scalar ( $S$ ) local operators of the form  $\mathcal{O}_\Gamma = \bar{q}\Gamma q$ , with  $\Gamma = \gamma_\mu\gamma_5$ ,  $i\sigma_{\mu\nu}$  and  $\mathbf{1}$ , respectively, whose matrix elements are evaluated in the forward limit. The general form for renormalized operators in the isovector flavour combination, at a scale  $\mu$ , reads

$$\mathcal{O}_\Gamma^{\overline{\text{MS}}}(\mu) = Z_{\mathcal{O}}^{\overline{\text{MS}},\text{Latt}}(\mu a, g^2) \left[ \mathcal{O}_\Gamma(a) + ab_{\mathcal{O}}m\mathcal{O}_\Gamma(a) + ac_{\mathcal{O}}\mathcal{O}_\Gamma^{\text{imp}}(a) \right] + \mathcal{O}(a^2), \quad (424)$$

where  $Z_{\mathcal{O}}^{\overline{\text{MS}},\text{Latt}}(\mu a, g^2)$  denotes the multiplicative renormalization factor determined in the chiral limit,  $m \rightarrow 0$ , and the second and third terms represent all possible quark-mass dependent and independent Symanzik improvement terms, respectively, at  $\mathcal{O}(a)$ .<sup>3</sup> The chiral properties of overlap, domain-wall fermions (with improvement up to  $\mathcal{O}(m_{\text{res}}^n)$  where  $m_{\text{res}}$  is the residual mass) and twisted-mass fermions (at maximal twist [105, 106]) mean that the  $\mathcal{O}(a)$  improvement terms are absent, while for nonperturbatively improved Sheikholeslami-Wohlert-Wilson (nonperturbatively-improved clover) fermions all terms appear in principle. For the operators of interest here there are several mass dependent terms but at most one dimension-four  $\mathcal{O}_\Gamma^{\text{imp}}$ ; see, e.g., Refs. [107, 108]. However, the latter involve external derivatives whose corresponding matrix elements vanish in the forward limit. Note that no mention is made of staggered fermions as they are not, currently, widely employed as valence quarks in nucleon matrix element calculations.

In order to illustrate the above remarks we consider the renormalization and improvement of the isovector axial current. This current has no anomalous dimension and hence the renormalization factor,  $Z_A = Z_A^{\overline{\text{MS}},\text{Latt}}(g^2)$ , is independent of the scale. The factor is usually computed nonperturbatively via the axial Ward identity [109] or the Rome-Southampton method [110] (see Sec. A.3 of FLAG 19 [61] for details). In some studies, the ratio with the corresponding vector renormalization factor,  $Z_A/Z_V$ , is determined for which some of the systematics cancel. In this case, one constructs the combination  $Z_{Ag_A}/(Z_Vg_V)$ , where  $Z_Vg_V = 1$  and  $g_A$  and  $g_V$  are the lattice forward matrix elements, to arrive at the renormalized axial charge [46]. For domain-wall fermions the ratio is employed in order to remove  $\mathcal{O}(am_{\text{res}})$  terms and achieve leading discretization effects starting at  $\mathcal{O}(a^2)$  [111]. Thus, as mentioned above,  $\mathcal{O}(a)$  improvement terms are only present for nonperturbatively-improved clover fermions. For the axial current, Eq. (424) takes the explicit form,

$$A_\mu^{\overline{\text{MS}}}(\mu) = Z_A^{\overline{\text{MS}},\text{Latt}}(g^2) \left[ \left( 1 + ab_A m_{\text{val}} + 3a\tilde{b}_A m_{\text{sea}} \right) A_\mu(a) + ac_A \partial_\mu P(a) \right] + \mathcal{O}(a^2), \quad (425)$$

where  $m_{\text{val}}$  and  $m_{\text{sea}}$  are the average valence- and sea-quark masses derived from the vector Ward identity [102, 108, 109], and  $P$  is the pseudoscalar operator  $\bar{q}\gamma_5 q$ . The matrix element of the derivative term is equivalent to  $q_\mu \langle N(p') | P | N(p) \rangle$  and hence vanishes in the forward limit when the momentum transfer  $q_\mu = 0$ . The improvement coefficients  $b_A$  and  $\tilde{b}_A$  are known perturbatively for a variety of gauge actions [107, 112, 113] and nonperturbatively for the tree-level Symanzik-improved gauge action for  $N_f = 2 + 1$  [114].

<sup>3</sup>Here  $a(g^2)$  refers to the lattice spacing in the chiral limit, however, lattice simulations are usually carried out by fixing the value of  $g^2$  while varying the quark masses. This means  $a = a(\tilde{g}^2)$  where  $\tilde{g}^2 = g^2(1 + b_g am_q)$  [101, 102] is the improved coupling that varies with the average sea-quark mass  $m_q$ . The difference between the  $Z$  factors calculated with respect to  $g^2$  and  $\tilde{g}^2$  can effectively be absorbed into the  $b_{\mathcal{O}}$  coefficients [103, 104].

Turning to operators for individual quark flavours, these can mix under renormalization and the singlet and nonsinglet renormalization factors can differ. For the axial current, such mixing occurs for all fermion formulations just like in the continuum, where the singlet combination acquires an anomalous dimension due to the  $U_A(1)$  anomaly. The ratio of singlet to nonsinglet renormalization factors,  $r_{\mathcal{O}} = Z_{\mathcal{O}}^s/Z_{\mathcal{O}}^{\text{n.s.}}$  for  $\mathcal{O} = A$  differs from 1 at  $\mathcal{O}(\alpha_s^2)$  in perturbation theory (due to quark loops), suggesting that the mixing is a small effect. The nonperturbative determinations performed so far find  $r_A \approx 1$  [5, 40], supporting this. For the tensor current the disconnected diagram vanishes in the continuum due to chirality and consequently on the lattice  $r_T = 1$  holds for overlap and DW fermions (assuming  $m_{\text{res}} = 0$  for the latter). For twisted-mass and clover fermions the mixing is expected to be small with  $r_T = 1 + \mathcal{O}(\alpha_s^3)$  [115] and this is confirmed by the nonperturbative studies of Refs. [42, 116].

The scalar operators for the individual quark flavours,  $\bar{q}q$ , are relevant not only for the corresponding scalar charges, but also for the sigma terms  $\sigma_q = m_q \langle N | \bar{q}q | N \rangle$  when combined with the quark masses  $m_q$ . For overlap and DW fermions  $r_S = 1$ , like in the continuum and all  $\bar{q}q$  renormalize multiplicatively with the isovector  $Z_S$ . The latter is equal to the inverse of the mass renormalization and hence  $m_q \bar{q}q$  is renormalization group (RG) invariant. For twisted-mass fermions, through the use of Osterwalder-Seiler valence fermions, the operators  $m_{ud}(\bar{u}u + \bar{d}d)$  and  $m_s \bar{s}s$  are also invariant [117].<sup>4</sup> In contrast, the lack of good chiral properties leads to significant mixing between quark flavours for clover fermions. Nonperturbative determinations via the axial Ward identity [35, 118] have found the ratio  $r_S$  to be much larger than the perturbative expectation  $1 + \mathcal{O}(\alpha_s^2)$  [115] may suggest. While the sum over the quark flavours which appear in the action  $\sum_q^{N_f} m_q \bar{q}q$  is RG invariant, large cancellations between the contributions from individual flavours can occur when evaluating, e.g., the strange sigma term. Note that for twisted-mass and clover fermions there is also an additive contribution  $\propto a^{-3} \mathbf{1}$  (or  $\propto \mu a^{-2} \mathbf{1}$ ) to the scalar operator. This contribution is removed from the nucleon scalar matrix elements by working with the subtracted current,  $\bar{q}q - \langle \bar{q}q \rangle$ , where  $\langle \bar{q}q \rangle$  is the vacuum expectation value of the current [108].

Symanzik improvement for the singlet currents follows the same pattern as in the isovector case with  $\mathcal{O}(a)$  terms only appearing for nonperturbatively-improved clover fermions. For the axial and tensor operators only mass dependent terms are relevant in the forward limit while for the scalar there is an additional gluonic operator  $\mathcal{O}_S^{\text{imp}} = \text{Tr}(F_{\mu\nu} F_{\mu\nu})$  with a coefficient of  $\mathcal{O}(\alpha_s)$  in perturbation theory. When constructing the sigma terms from the quark masses and the scalar operator, the improvement terms remain and they must be included to remove all  $\mathcal{O}(a)$  effects for nonperturbatively-improved clover fermions, see Ref. [108] for a discussion.

#### 10.1.4 Extrapolations in $a$ , $M_\pi$ and $M_\pi L$

To obtain physical results that can be used to compare to or make predictions for experiment, all quantities must be extrapolated to the continuum and infinite-volume limits. In general, either a chiral extrapolation or interpolation must also be made to the physical pion mass. These extrapolations need to be performed simultaneously since discretization and finite-volume effects are themselves dependent upon the pion mass. Furthermore, in practice it is not possible to hold the pion mass fixed while the lattice spacing is varied, as some variation

<sup>4</sup>Note that for twisted-mass fermions the pseudoscalar renormalization factor is the relevant factor for the scalar operator. The isovector (isosinglet) scalar current in the physical basis becomes the isosinglet (isovector) pseudoscalar current in the twisted basis. Perturbatively  $r_P = 1 + \mathcal{O}(\alpha_s^3)$  and nonperturbative determinations have found  $r_P \approx 1$  [42].

in  $a$  occurs when tuning the quark masses at fixed gauge coupling. Thus, one performs a simultaneous extrapolation in all three variables using a theoretically motivated formula of the form,

$$g(M_\pi, a, L) = g_{\text{phys}} + \delta_{M_\pi} + \delta_a + \delta_L, \quad (426)$$

where  $g_{\text{phys}}$  is the desired extrapolated result, and  $\delta_{M_\pi}$ ,  $\delta_a$ ,  $\delta_L$  are the deviations due to the pion mass, the lattice spacing, and the volume, respectively. Below we outline the forms for each of these terms.

All observables discussed in this section are dimensionless, therefore the extrapolation formulae may be parameterized by a set of dimensionless variables:

$$\epsilon_\pi = \frac{M_\pi}{\Lambda_\chi}, \quad M_\pi L, \quad \epsilon_a = \Lambda_a a. \quad (427)$$

Here,  $\Lambda_\chi \sim 1$  GeV is a chiral symmetry breaking scale, which, for example, can be set to  $\Lambda_\chi = 4\pi F_\pi$ , where  $F_\pi = 92.2$  MeV is the pion decay constant, and  $\Lambda_a$  is a discretization scale, e.g.,  $\Lambda_a = \frac{1}{4\pi w_0}$ , where  $w_0$  is a gradient-flow scale [119].

Effective field theory methods may be used to determine the form of each of these extrapolations. For the single nucleon charges, Heavy-Baryon  $\chi$ PT (HB $\chi$ PT) is a common choice [120, 121], however, other variants, such as unitarized [122] or covariant  $\chi$ PT [123, 124], are also employed. Various formulations of HB $\chi$ PT exist, including those for two- and three-flavours, as well as with and without explicit  $\Delta$  baryon degrees of freedom. Two-flavour HB $\chi$ PT is typically used due to issues with convergence of the three-flavour theory [37, 125–128]. The convergence properties of all known formulations for baryon  $\chi$ PT, even at the physical pion mass, have not been well-established.

To  $\mathcal{O}(\epsilon_\pi^2)$ , the two-flavour chiral expansion for the nucleon charges is known to be of the form [129],

$$g = g_0 + g_1 \epsilon_\pi + g_2 \epsilon_\pi^2 + \tilde{g}_2 \epsilon_\pi^2 \ln(\epsilon_\pi^2), \quad (428)$$

where  $g_1 = 0$  for all charges  $g$  except  $g_S^{u,d}$ . The dimensionless coefficients  $g_{0,1,2}, \tilde{g}_2$  are assumed to be different for each of the different charges. The coefficients in front of the logarithms,  $\tilde{g}_2$ , are known functions of the low-energy constants (LECs), and do not represent new, independent LECs. Mixed action calculations will have further dependence upon the mixed valence-sea pion mass,  $m_{vS}$ .

Given the potential difficulties with convergence of the chiral expansion, known values of the  $\tilde{g}_2$  in terms of LECs are not typically used, but are left as free fit parameters. Furthermore, many quantities have been found to display mild pion mass dependence, such that Taylor expansions, i.e., neglecting logarithms in the above expressions, are also often employed. The lack of a rigorously established theoretical basis for the extrapolation in the pion mass thus requires data close to the physical pion mass for obtaining high precision extrapolated/interpolated results.

Discretization effects depend upon the lattice action used in a particular calculation, and their form may be determined using the standard Symanzik power counting. In general, for an unimproved action, the corrections due to discretization effects  $\delta_a$  include terms of the form,

$$\delta_a = c_1 \epsilon_a + c_2 \epsilon_a^2 + \dots, \quad (429)$$

where  $c_{1,2}$  are dimensionless coefficients. Additional terms of the form  $\tilde{c}_n (\epsilon_\pi \epsilon_a)^n$ , where  $n$  is an integer whose lowest value depends on the combined discretization and chiral properties, will also appear. Improved actions systematically remove correction terms, e.g., an  $\mathcal{O}(a)$ -improved action, combined with a similarly improved operator, will contain terms in the extrapolation ansatz beginning at  $\epsilon_a^2$  (see Sec. 10.1.3).

Finite volume corrections  $\delta_L$  may be determined in the usual way from effective field theory, by replacing loop integrals over continuous momenta with discrete sums. Finite volume effects therefore introduce no new undetermined parameters to the extrapolation. For example, at next-to-leading order, and neglecting contributions from intermediate  $\Delta$  baryons, the finite-volume corrections for the axial charge in two-flavour HB $\chi$ PT take the form [130],

$$\delta_L \equiv g_A(L) - g_A(\infty) = \frac{8}{3} \epsilon_\pi^2 \left[ g_0^3 F_1(M_\pi L) + g_0 F_3(M_\pi L) \right], \quad (430)$$

where

$$\begin{aligned} F_1(mL) &= \sum_{\mathbf{n} \neq \mathbf{0}} \left[ K_0(mL|\mathbf{n}|) - \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|} \right], \\ F_3(mL) &= -\frac{3}{2} \sum_{\mathbf{n} \neq \mathbf{0}} \frac{K_1(mL|\mathbf{n}|)}{mL|\mathbf{n}|}, \end{aligned} \quad (431)$$

and  $K_\nu(z)$  are the modified Bessel functions of the second kind. Some extrapolations are performed using the form for asymptotically large  $M_\pi L$ ,

$$K_0(z) \rightarrow \frac{e^{-z}}{\sqrt{z}}, \quad (432)$$

and neglecting contributions due to  $K_1$ . Care must, however, be taken to establish that these corrections are negligible for all included values of  $M_\pi L$ . The numerical coefficients, for example,  $8/3$  in Eq. (430), are often taken to be additional free fit parameters, due to the question of convergence of the theory discussed above.

Given the lack of knowledge about the convergence of the expansions and the resulting plethora of possibilities for extrapolation models at differing orders, it is important to include statistical tests of model selection for a given set of data. Bayesian model averaging [131] or use of the Akaike Information Criterion [132] are common choices which penalize over-parameterized models.

## 10.2 Quality criteria for nucleon matrix elements and averaging procedure

There are two specific issues that call for a modification and extension of the FLAG quality criteria listed in Sec. 2. The first concerns the rating of the chiral extrapolation: The FLAG criteria reflect the ability of  $\chi$ PT to provide accurate descriptions of the pion mass dependence of observables. Clearly, this ability is linked to the convergence properties of  $\chi$ PT in a particular mass range. Quantities extracted from nucleon matrix elements are extrapolated to the physical pion mass using some variant of baryonic  $\chi$ PT, whose convergence is not well established as compared to the mesonic sector. Therefore, we have opted for stricter quality criteria,  $200 \text{ MeV} \leq M_{\pi, \text{min}} \leq 300 \text{ MeV}$ , for a green circle in the chiral extrapolation of

nucleon matrix elements, i.e.,

- ★  $M_{\pi,\min} < 200$  MeV with three or more pion masses used in the extrapolation  
or two values of  $M_{\pi}$  with one lying within 10 MeV of 135 MeV (the physical neutral pion mass) and the other one below 200 MeV
- $200 \text{ MeV} \leq M_{\pi,\min} \leq 300$  MeV with three or more pion masses used in the extrapolation;  
or two values of  $M_{\pi}$  with  $M_{\pi,\min} < 200$  MeV;  
or a single value of  $M_{\pi}$  lying within 10 MeV of 135 MeV (the physical neutral pion mass)
- Otherwise

In Sec. 10.1.2 we have discussed that insufficient control over excited-state contributions, arising from the noise problem in baryonic correlation functions, may lead to a systematic bias in the determination of nucleon matrix elements. We therefore introduce an additional criterion that rates the efforts to suppress excited-state contamination in the final result. As described in Sec. 10.1.2, the applied methodology to control excited-state contamination is quite diverse. Since a broad consensus on the question which procedures should be followed has yet to emerge, our criterion is expressed in terms of simulation parameters that can be straightforwardly extracted on the basis of publications. Furthermore, the criterion must also be readily applicable to a variety of different local operators whose matrix elements are discussed in this chapter. These requirements are satisfied by the source-sink separation  $\tau$ , i.e., the Euclidean distance between the initial and final nucleons. The discussion at the end of Sec. 10.1.2 shows that there is room for improvement in the ability to control excited-state contamination. Hence, we have reverted to a binary system, based on the range of source-sink separations of a given calculations. While we do not award the highest category—a green star—in this edition, we stress that the adoption of the modified ESC criterion has not led to a situation where calculations that were previously rated with a green star are now excluded from FLAG averages. The rating scale concerning control over excited-state contributions is thus

- Three or more source-sink separations  $\tau$ , at least two of which must be above 1.0 fm.
- Otherwise

We will continue to monitor the situation concerning excited-state contamination and, if necessary, adapt the criteria further in future editions of the FLAG report.

As explained in Sec. 2, FLAG averages are distinguished by the sea-quark content. Hence, for a given configuration of the quark sea (i.e., for  $N_f = 2, 2+1, 2+1+1$ , or  $1+1+1+1$ ), we first identify those calculations that pass the FLAG and the additional quality criteria defined in this section, i.e. excluding any calculation that has a red tag in one or more of the categories. We then add statistical and systematic errors in quadrature and perform a weighted average. If the fit is of bad quality (i.e., if  $\chi_{\min}^2/\text{dof} > 1$ ), the errors of the input quantities are scaled by  $\sqrt{\chi^2/\text{dof}}$ . In the following step, correlations among different calculations are taken into account in the error estimate by applying Schmelling’s procedure [133].

### 10.3 Isovector charges

The axial, scalar and tensor isovector charges are needed to interpret the results of many experiments and phenomena mediated by weak interactions, including probes of new physics.

The most natural process from which isovector charges can be measured is neutron beta decay ( $n \rightarrow p^+ e^- \bar{\nu}_e$ ). At the quark level, this process occurs when a down quark in a neutron transforms into an up quark due to weak interactions, in particular due to the axial current interaction. While scalar and tensor currents have not been observed in nature, effective scalar and tensor interactions arise in the SM due to loop effects. At the TeV and higher scales, contributions to these three currents could arise due to new interactions and/or loop effects in BSM theories. These super-weak corrections to standard weak decays can be probed through high precision measurements of the neutron decay distribution by examining deviations from SM predictions as described in Ref. [134]. The lattice-QCD methodology for the calculation of isovector charges is well-established, and the control over statistical and systematic uncertainties is becoming robust.

The axial charge  $g_A^{u-d}$  is an important parameter that encapsulates the strength of weak interactions of nucleons. It enters in many analyses of nucleon structure and of SM and BSM physics. For example, it enters in (i) the extraction of  $V_{ud}$  and tests of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix; (ii) the analysis of neutrinoless double-beta decay, (iii) neutrino-nucleus quasi-elastic scattering cross-section; (iv) the rate of proton-proton fusion, the first step in the thermonuclear reaction chains that power low-mass hydrogen-burning stars like the Sun; (v) solar and reactor neutrino fluxes; (vi) muon capture rates, etc.. The current best determination of the ratio of the axial to the vector charge,  $g_A/g_V$ , comes from measurement of neutron beta decay using polarized ultracold neutrons by the UCNA collaboration,  $1.2772(20)$  [135, 136], and by PERKEO II,  $1.2761_{-17}^{+14}$  [137]. Note that, in the SM,  $g_V = 1$  up to second-order corrections in isospin breaking [138, 139] as a result of the conservation of the vector current. Given the accuracy with which  $g_A^{u-d}$  has been measured in experiments, the goal of lattice-QCD calculations is to calculate it directly with  $\mathcal{O}(1\%)$  accuracy.

Isovector scalar or tensor interactions contribute to the helicity-flip parameters, called  $b$  and  $B$ , in the neutron decay distribution. By combining the calculation of the scalar and tensor charges with the measurements of  $b$  and  $B$ , one can put constraints on novel scalar and tensor interactions at the TeV scale as described in Ref. [134]. To optimally bound such scalar and tensor interactions using measurements of  $b$  and  $B$  parameters in planned experiments targeting  $10^{-3}$  precision [140–142], we need to determine  $g_S^{u-d}$  and  $g_T^{u-d}$  at the 10% level as explained in Refs. [46, 134]. Future higher-precision measurements of  $b$  and  $B$  would require correspondingly higher-precision calculations of the matrix elements to place even more stringent bounds on these couplings at the TeV-scale.

One can estimate  $g_S^{u-d}$  using the conserved vector current (CVC) relation,  $g_S/g_V = (M_{\text{neutron}} - M_{\text{proton}})^{\text{QCD}} / (m_d - m_u)^{\text{QCD}}$ , as done by Gonzalez-Alonso *et al.* [143]. In their analysis, they took estimates of the two mass differences on the right-hand side from the global lattice-QCD data [144] and obtained  $g_S^{u-d} = 1.02(8)(7)$ .

The tensor charge  $g_T^{u-d}$  can be extracted experimentally from semi-inclusive deep-inelastic scattering (SIDIS) data [145–148]. A sample of these phenomenological estimates is shown in Fig. 45, and the noteworthy feature is that the current uncertainty in these phenomenological estimates is large.

### 10.3.1 Results for $g_A^{u-d}$

Calculations of the isovector axial charge have a long history, as can be seen from the compilation given in Tab. 69 and plotted in Fig. 43. The issue of excited-state contamination received

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$g_A^{u-d}$
CalLat 19	[149]	2+1+1	C	○	★	★	★	○	1.2642(93)
ETM 19	[150]	2+1+1	A	■	○	★	★	○	1.286(23)
PNDME 18 <sup>a</sup>	[50]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	1.218(25)(30)
CalLat 18	[51]	2+1+1	A	○	★	★	★	○	1.271(10)(7)
CalLat 17	[47]	2+1+1	P	○	★	★	★	○	1.278(21)(26)
PNDME 16 <sup>a</sup>	[46]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	1.195(33)(20)
NME 21 <sup>a</sup>	[151]	2+1	P	○ <sup>‡</sup>	★	★	★	○	1.31(6)(5)
LHPC 19	[13]	2+1	A	■ <sup>‡</sup>	★	★	★	○	1.265(49)
Mainz 19	[84]	2+1	A	★	○	★	★	○	1.242(25)( $^{+0}_{-0.030}$ )
PACS 18A	[11]	2+1	A	■	★	★	★	○	1.273(24)(5)(9)
PACS 18	[9]	2+1	A	■	■	★	★	■	1.163(75)(14)
$\chi$ QCD 18	[26]	2+1	A	○	★	★	★	○	1.254(16)(30) <sup>§</sup>
JLQCD 18	[60]	2+1	A	■	○	○	★	○	1.123(28)(29)(90)
LHPC 12A <sup>b</sup>	[152]	2+1	A	■ <sup>‡</sup>	★	★	★	○	0.97(8)
LHPC 10	[68]	2+1	A	■	○	■	★	■	1.21(17)
RBC/UKQCD 09B	[53]	2+1	A	■	■	○	★	■	1.19(6)(4)
RBC/UKQCD 08B	[52]	2+1	A	■	■	○	★	■	1.20(6)(4)
LHPC 05	[153]	2+1	A	■	■	★	★	■	1.226(84)
Mainz 17	[36]	2	A	★	★	★	★	■	1.278(68)( $^{+0}_{-0.087}$ )
ETM 17B	[40]	2	A	■	○	○	★	○	1.212(33)(22)
ETM 15D	[38]	2	A	■	○	○	★	○	1.242(57)
RQCD 14	[34]	2	A	○	★	★	★	■	1.280(44)(46)
QCDSF 13	[32]	2	A	○	★	■	★	■	1.29(5)(3)
Mainz 12	[33]	2	A	★	○	○	★	■	1.233(63)( $^{+0.035}_{-0.060}$ )
RBC 08	[154]	2	A	■	■	■	★	■	1.23(12)
QCDSF 06	[30]	2	A	○	■	■	★	■	1.31(9)(7)

<sup>a</sup> The improvement coefficient in the valence quark action is set to its tadpole-improved tree-level value.

<sup>b</sup> The quark action is tree-level improved.

<sup>‡</sup> The rating takes into account that the action is not fully  $O(a)$  improved by requiring an additional lattice spacing.

<sup>§</sup> For this partially quenched analysis the criteria are applied to the unitary points.

Table 69: Overview of results for  $g_A^{u-d}$ .

little if any attention before 2010. As a consequence, the range of source-sink separations employed in many of the early calculations prior to that year was rather limited, offering little control over this important systematic effect. This concerns, in particular, the calculations by LHPC 05 [153], LHPC 10 [68], RBC 08 [154], RBC/UKQCD 08B [52], RBC/UKQCD 09B [53] and QCDSF 06 [30]. Since the last edition of the FLAG report, no new results in two-flavour QCD have been published. An exception is the calculation ETM 19 [150], which reanalyzed two ensembles with  $N_f = 2$  around the physical pion mass to study finite-volume effects, while the main result is quoted from a calculation with  $N_f = 2 + 1 + 1$ . These two-flavour calculations still do not qualify for inclusion in the FLAG average (see Table 69). We thus refrain from providing a detailed discussion of the results in Refs. [30, 32–34, 36, 38, 40, 154] and refer the reader to the corresponding chapter in the previous edition of the FLAG report.

Estimates for the axial charge with  $N_f = 2 + 1$  have been published by many collaborations, i.e., LHPC [13, 68, 152, 153], RBC/UKQCD [52, 53], JLQCD 18 [60],  $\chi$ QCD 18 [26], PACS 18/PACS 18A [9, 11], Mainz 19 [84] (superseding the previously listed result in [155]) and NME 21 [151].

The calculations in LHPC 05 [153] and LHPC 10 [68] were based on a mixed-action setup, combining domain-wall fermions in the valence sector with staggered (asqtad) gauge ensembles generated by MILC. Although the dependence of the results on the source-sink separation was studied to some extent in LHPC 10, excited-state effects are not sufficiently controlled according to our quality criteria described in Sec. 10.2. A different discretization of the quark action was used in their later studies (LHPC 12A [152] and LHPC 19 [13]), employing tree-level improved Wilson fermions with smeared gauge links, both in the sea and valence sectors. While this setup does not realize full  $\mathcal{O}(a)$  improvement, it was found that smeared gauge links reduce the leading discretization effects of  $\mathcal{O}(a)$  substantially. The most recent publication (LHPC 19) is based on two ensembles within 1.5% of the physical pion, at two different values of the lattice spacing. Results for  $g_A^{u-d}$  were determined using the summation and ratio methods, with and without including the first excitation in the fit. LHPC quotes the result from the finer lattice spacing, with an error that covers the spread of uncertainties on both ensembles.

The RBC/UKQCD collaboration has employed  $N_f = 2 + 1$  flavours of domain-wall fermions. The results quoted in RBC/UKQCD 08B [52] and RBC/UKQCD 09B [53] were obtained at relatively heavy pion masses at a single value of the lattice spacing, with only limited control over excited-state effects. While systematic investigation of different source-sink separations has been recently performed on two ensembles at the same lattice spacing and pion masses of 250 and 170 MeV, respectively [156], an estimate for  $g_A^{u-d}$  at the physical point has not been quoted.

The JLQCD collaboration (JLQCD 18 [60]) has performed a calculation using  $N_f = 2 + 1$  flavours of overlap fermions and the Iwasaki gauge action. Owing to the large numerical cost of overlap fermions, which preserve exact chiral symmetry at nonzero lattice spacing, they have only simulated four light quark masses with  $290 < M_\pi < 540$  MeV and at a single lattice spacing so far. Their simultaneous fit to the data for the correlator ratio  $R_A(t, \tau)$  computed at six values of  $\tau$  to a constant, gives a low value for  $g_A^{u-d}$  at the physical point. Overlap valence quarks were also used by the  $\chi$ QCD collaboration in their study of various nucleon matrix elements ( $\chi$ QCD 18 [26]), utilizing the gauge ensembles generated by RBC/UKQCD with domain-wall fermions. The quoted estimate for the axial charge was obtained from a combination of two-state fits and the summation method, applied over a range of source-sink separations.

Calculations with  $N_f = 2 + 1$  flavours of  $\mathcal{O}(a)$  improved Wilson fermions have been performed by PACS, the Mainz group and NME. The calculations by the PACS collaboration (PACS 18 [9] and PACS 18A [11]) were performed on very large volumes (8.2 fm and 10.8 fm, respectively) at or near the physical pion mass. In PACS 18A, the ratio method without including excited states was used to determine the isovector axial charge, which was found to be in good agreement with the experimental value. However, only a single lattice spacing was used in PACS 18 and PACS 18A, so that these calculations lack control over discretization effects. The Mainz group (Mainz 19 [84]) has presented results for the axial charge, obtained by performing two-state fits to six different nucleon matrix elements (including the scalar and tensor charges), assuming that the mass gap to the excited state can be more reliably constrained in this way. Up to six source-sink separations per ensemble have been studied. The final results are obtained from a combined chiral, continuum and finite-volume extrapolation. The NME collaboration (NME 21 [151]) has recently published the results from a calculation of various nucleon form factors and charges. Results were obtained from multi-state fits, using up to four (three) states in the two-point (three-point) correlation functions. In order to describe and control excited-state effects,  $N\pi$  and  $N\pi\pi$  states with different relative momenta were included in the analysis. The preferred result for  $g_A^{u-d}$  was obtained from the axial form factor  $G_A(Q^2)$  extrapolated to  $Q^2 = 0$ .

Three groups, PNDME, CalLat and ETMC, have published results for  $N_f = 2 + 1 + 1$ , i.e. PNDME 16 [46], PNDME 18 [50], CalLat 17 [47] CalLat 18 [51], CalLat 19 [149]. PNDME and CalLat share the staggered (HISQ) gauge ensembles generated by the MILC collaboration, but employ different discretizations in the valence quark sector: PNDME use  $\mathcal{O}(a)$  improved Wilson fermions with the improvement coefficient  $c_{\text{sw}}$  set to its tadpole-improved tree-level value. By contrast, CalLat use the Möbius variant of domain-wall fermions, which are fully  $\mathcal{O}(a)$  improved. The CalLat set of ensembles includes three values of the lattice spacing, i.e.  $a = 0.09, 0.12,$  and  $0.15$  fm, while PNDME added another set of ensembles at the finer lattice spacing of  $0.06$  fm to this collection. Both groups have included physical pion mass ensembles in their calculations. The operator matrix elements are renormalized nonperturbatively, using the Rome-Southampton method.

In order to control excited-state contamination, PNDME perform multi-state fits, including up to four (three) energy levels in the two-point (three-point) correlation functions. By contrast, CalLat have employed the Feynman-Hellmann-inspired implementation of summed operator insertions described in Sec. 10.1.2. Plotting the summed correlator  $S_A(\tau)$  as a function of the source-sink separation, they find that excited-state effects cannot be detected for  $\tau \gtrsim 1.0$  fm at their level of statistics. After subtracting the leading contributions from excited states determined from two-state fits, they argue that the data for  $S_A(\tau)$  can be described consistently down to  $\tau \simeq 0.3$  fm.

The recent calculation by ETMC (ETM 19 [150]) with  $N_f = 2 + 1 + 1$  was performed using a single twisted-mass QCD ensemble with  $m_\pi \approx 139$  MeV. In order to control excited-state effects, the summation method and multi-state fits were used. No significant finite-volume effects were expected based on a similar analysis of two  $N_f = 2$  ensembles with different spatial extents. The quoted estimate is identified with the result obtained from a two-state fit on the single  $N_f = 2 + 1 + 1$  ensemble, which agrees with the value determined from the summation method.

We now proceed to discuss global averages for the axial charge, in accordance with the procedures in Sec. 10.2. For QCD with  $N_f = 2 + 1 + 1$ , the calculations of PNDME and CalLat pass all our quality criteria, while the result of ETM 19 is excluded due to the fact that it was

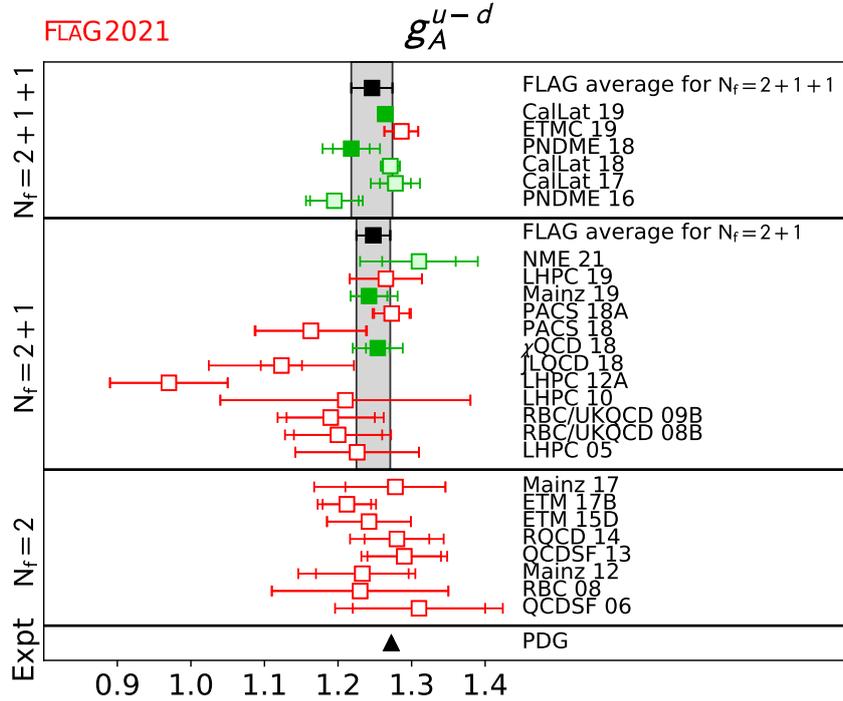


Figure 43: Lattice results and FLAG averages for the isovector axial charge  $g_A^{u-d}$  for  $N_f = 2, 2 + 1$  and  $2 + 1 + 1$  flavour calculations. Also shown is the experimental result as quoted in the PDG [157].

performed at a single value of the lattice spacing. Hence the results from PNDME 18 [50] and CalLat 19 [149], which is an update of CalLat 18 [51], qualify for being included in a global average. Since both PNDME and CalLat use gauge ensembles produced by MILC, we assume that the quoted errors are 100% correlated, even though the range of pion masses and lattice spacings explored in Refs. [50] and [51, 149] is not exactly identical. Performing a weighted average yields  $g_A^{u-d} = 1.2617(126)$  with  $\chi^2/\text{dof} = 1.33$ , where the error has been scaled by about 15% because of the large  $\chi^2/\text{dof}$ . The result by CalLat dominates the weighted average due to its smaller error. Given that the calculations of PNDME 18 and CalLat 19 are correlated, the large value of  $\chi^2/\text{dof}$  indicates a slight tension between the two results. In this situation we adopt a more conservative approach, by requiring that the uncertainty assigned to the FLAG estimate encompasses the central value of PNDME 18. As a result, we choose to represent the axial charge by the interval  $1.218 \leq g_A^{u-d} \leq 1.274$ , where the lower bound is identified with the result of PNDME 18, while the upper bound is the weighted average plus the scaled  $1\sigma$  uncertainty. Hence, for  $N_f = 2 + 1 + 1$  we quote  $g_A^{u-d} = 1.246(28)$  as the FLAG estimate, where the central value marks the mid-point of the interval, and half the width is taken to be the error.

For QCD with  $N_f = 2 + 1$  dynamical quarks, the calculations of  $\chi$ QCD 18 [26], Mainz 19 [84] and NME 21 [151] are free of red tags, while the calculation by PACS 18A [11] and LHPC 19 [13] do not offer enough control over lattice artefacts according to the FLAG criteria. Since the result by NME 21 was published only as a preprint by the FLAG deadline, it does not qualify for being included in a global average. Hence, for  $N_f = 2 + 1$  we compute a weighted average from  $\chi$ QCD 18 [26] and Mainz 19 [84], assuming no correlations between the two

calculations. This yields  $g_A^{u-d} = 1.248(23)$  with  $\chi^2/\text{dof} = 0.07$ .

Due to the modified criteria for excited-state contamination, none of the results obtained in two-flavour QCD qualify for a global average. Nonetheless, we find it instructive to show the results for  $N_f = 2$  together with the calculations with  $N_f = 2 + 1$  and  $2 + 1 + 1$  and the respective FLAG estimates in Fig. 43.

To summarize, the FLAG averages for the axial charge read

$$N_f = 2 + 1 + 1 : \quad g_A^{u-d} = 1.246(28) \quad \text{Refs. [50, 51, 149]}, \quad (433)$$

$$N_f = 2 + 1 : \quad g_A^{u-d} = 1.248(23) \quad \text{Ref. [26, 84]}, \quad (434)$$

Within errors, these averages are compatible with the result of  $g_A^{u-d} = 1.2724(23)$  quoted by the PDG. While the most recent lattice calculations reproduce the axial charge at the level of a few percent or even better, the experimental result is more precise by an order of magnitude.

### 10.3.2 Results for $g_S^{u-d}$

Calculations of the isovector scalar charge have, in general, larger errors than the isovector axial charge as can be seen from the compilation given in Tab. 70 and plotted in Fig. 44. The isovector scalar charge can also be determined indirectly via the conserved vector current (CVC) relation from results for the neutron-proton mass difference [160–168] and the down and up quark mass difference (see Sec. 3.1.6). For comparison, Fig. 44 also shows an indirect determination obtained using lattice and phenomenological input [143].

As in FLAG 19, for 2+1+1 flavours, only PNDME 18 [50], which supersedes PNDME 16 [46] and PNDME 13 [43], meets all the criteria for inclusion in the average. The discussions for this and other past calculations are repeated from FLAG 19 for completion.

This mixed-action calculation was performed using the MILC HISQ ensembles, with a clover valence action. The 11 ensembles used include three pion mass values,  $M_\pi \sim 135, 225, 320$  MeV, and four lattice spacings,  $a \sim 0.06, 0.09, 0.12, 0.15$  fm. Note that four lattice spacings are required to meet the green star criteria, as this calculation is not fully  $\mathcal{O}(a)$  improved. Lattice size ranges between  $3.3 \lesssim M_\pi L \lesssim 5.5$ . Physical point extrapolations were performed simultaneously, keeping only the leading-order terms in the various expansion parameters. For the finite-volume extrapolation, the asymptotic limit of the  $\chi$ PT prediction, Eq. (432), was used. Excited-state contamination is controlled using two-state fits to between three and five source-sink time separations between  $0.72 \lesssim \tau \lesssim 1.68$  fm. Renormalization was performed nonperturbatively using the RI-SMOM scheme and converted to  $\overline{\text{MS}}$  at 2 GeV using 2-loop perturbation theory.

The calculation performed in ETM 19 [150] was generated using twisted-mass fermions with a clover term. The calculation utilized a single 2+1+1-flavour gauge configuration, with a pion mass near the physical point,  $m_\pi \sim 139$  MeV, lattice spacing of  $a \sim 0.08$  fm, and volume corresponding to  $m_\pi L = 3.86$ . Seven source-sink separations were used in the analysis, ranging from  $t = 0.64$ –1.6 fm. Two further two-flavour ensembles were also explored, having the same pion mass,  $m_\pi \sim 130$  MeV and lattice spacing  $a \sim 0.09$  fm, but with different volumes corresponding to  $m_\pi L \sim 3$  and  $m_\pi L \sim 4$ . The final result is quoted from the single 2+1+1 flavour ensemble and does not include an assessment of discretization systematics, and therefore does not meet the continuum quality criterion for inclusion in the average.

Regarding 2+1-flavour calculations, a single calculation meets all criteria necessary for inclusion in the average. The Mainz 19 [84] calculation was performed on the Wilson CLS

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$g_S^{u-d}$
ETM 19	[150]	2+1+1	A	■	○	★	★	○	1.35(17)
PNDME 18	[50]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	1.022(80)(60)
PNDME 16	[46]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	0.97(12)(6)
PNDME 13	[43]	2+1+1	A	■ <sup>‡</sup>	■	★	★	○	0.72(32)
NME 21	[151]	2+1	P	○ <sup>‡</sup>	★	★	★	○	1.06(10)(6)
$\chi$ QCD 21A	[158]	2+1	P	★	★	★	★	○	0.94(10)(6)
RBC/UKQCD 19	[156]	2+1	A	■	○	★	★	■	0.9(3)
Mainz 19	[84]	2+1	A	★	○	★	★	○	1.13(11)( $\bar{6}$ )
LHPC 19	[13]	2+1	A	■ <sup>‡</sup>	★	★	★	○	0.927(303)
JLQCD 18	[60]	2+1	A	■	○	○	★	○	0.88(8)(3)(7)
LHPC 12	[159]	2+1	A	■ <sup>‡</sup>	★	★	★	○	1.08(28)(16)
ETM 17	[42]	2	A	■	○	○	★	○	0.930(252)(48)(204)
RQCD 14	[34]	2	A	○	★	★	★	■	1.02(18)(30)

<sup>‡</sup> The rating takes into account that the action is not fully  $O(a)$  improved by requiring an additional lattice spacing.

Table 70: Overview of results for  $g_S^{u-d}$ .

ensembles, using four lattice spacings ( $a \sim 0.05$  fm to 0.086 fm), several pion masses ranging from  $\sim 200$  MeV to  $\sim 350$  MeV, and volumes corresponding to  $m_\pi L \sim 3$  to  $\sim 5.4$ . Physical point extrapolations were performed simultaneously in the lattice spacing, pion mass, and volume. Excited states were controlled using two-state simultaneous fits to multiple observables, and included several source-sink separations typically in the range 1-1.5 fm. Renormalization was performed nonperturbatively using the RI-SMOM scheme and converted to  $\overline{\text{MS}}$  at 2 GeV using 2-loop perturbation theory.

The 2+1-flavour calculation of  $\chi$ QCD 21A [158] was performed using a mixed-action approach with domain-wall fermion gauge configurations generated by the RBC/UKQCD collaboration and overlap valence quarks. They include five pion masses ranging from  $m_\pi \sim 140$  MeV to 370 MeV, four lattice spacings ( $a \sim 0.06, 0.08, 0.11,$  and 0.14 fm). Three to six different valence-quark masses are computed on each ensemble. The extrapolation to the physical pion mass, continuum and infinite-volume limits is obtained by a global fit of all data to a partially quenched chiral perturbation theory ansatz. Excited-state contamination is assessed using three to five sink-source separations and multi-state fits. Renormalization is

performed using RI/MOM and the final result quoted in  $\overline{\text{MS}}$  at 2 GeV. At the time of writing of this review, this calculation was unpublished and the results are therefore not included in the average.

The NME 21[151] 2+1-flavour calculation utilized seven ensembles of Wilson-clover fermions. Three lattice spacings, ranging from  $a \sim 0.07$  fm to 0.13 fm, several pion masses,  $m_\pi \sim 165$  MeV to 285 MeV, and volumes corresponding to  $m_\pi L \sim 3.75$  to 6.15 were used. Combined continuum, chiral, and infinite-volume extrapolations are performed to the physical point using leading-order fit functions. Several fitting strategies are explored using four to six source-sink separations ranging from 0.7–1.8 fm. Final results are quoted by averaging results from two of these fitting strategies, in which the excited-state energy for the three-point function is fixed using two different prior strategies. Renormalization is non-perturbative (RI-SMOM) using two strategies, and quoted in  $\overline{\text{MS}}$  at 2 GeV. This work was also unpublished at the time of writing of this review and is not included in the average.

The RBC/UKQCD 19 [156] calculation employed 2+1 flavours of domain-wall fermions using an Iwasaki and dislocation-suppressing-determinant-ratio gauge action. They utilized two values of the pion mass,  $m_\pi \sim 250$  and 170 MeV with volumes corresponding to  $m_\pi L \sim 5.8$  and 4.0, respectively. The results are quoted using only one lattice spacing of 0.14 fm, and a single source-sink separation of 1.3 fm and therefore do not meet the criteria for continuum or excited-state contamination. The LHPC 19 [13] calculation used a 2+1 flavour 2-HEX-smear Wilson-clover action with two ensembles near the physical pion mass,  $m_\pi \sim 133$  and 137 MeV. The lattice spacings corresponded to  $a \sim 0.09$  and 0.12 fm and volumes  $m_\pi L \sim 4$ . They used 3 and 8 different time separations for the two ensembles and compare ratio, summation, and multi-state methods to assess excited-state contamination. Because the calculation is not fully  $\mathcal{O}(a)$  improved, an additional lattice spacing would be necessary to meet the continuum criterion for inclusion in the average.

The JLQCD 18 [60] calculation, performed using overlap fermions on the Iwasaki gauge action, covered four pion masses down to 290 MeV. The lattice size was adjusted to keep  $M_\pi L \geq 4$  in all four cases. However, the single lattice spacing of  $a = 0.11$  fm does not meet the criteria for continuum extrapolation. The calculations presented in LHPC 12A used three different lattice actions, Wilson-clover, domain-wall, and mixed action. Pion masses ranged down to near the physical pion mass. Data at two lattice spacings were produced with the domain-wall and Wilson actions, however, the final result utilized only the single lattice spacing of  $a = 0.116$  fm from the Wilson action. Because the action is not fully  $\mathcal{O}(a)$  improved, two lattice spacings are not sufficient for meeting the quality criteria for the continuum extrapolation.

The two-flavour calculations in Tab. 70 include ETM 17, which employed twisted-mass fermions on the Iwasaki gauge action<sup>5</sup>. This work utilized a single physical pion mass ensemble with lattice spacing  $a \sim 0.09$  fm, and therefore does not meet the criteria for continuum extrapolation. The RQCD 14 calculation included three lattice spacings down to 0.06 fm and several pion masses down to near the physical point. While a study of excited-state contamination was performed on some ensembles using multiple source-sink separations, many ensembles included only a single time separation, so it does not meet the criteria for excited states.

<sup>5</sup>The earlier work, ETM 15D [38], did not give a final value for  $g_S^{u-d}$  and is therefore not included in the tables.

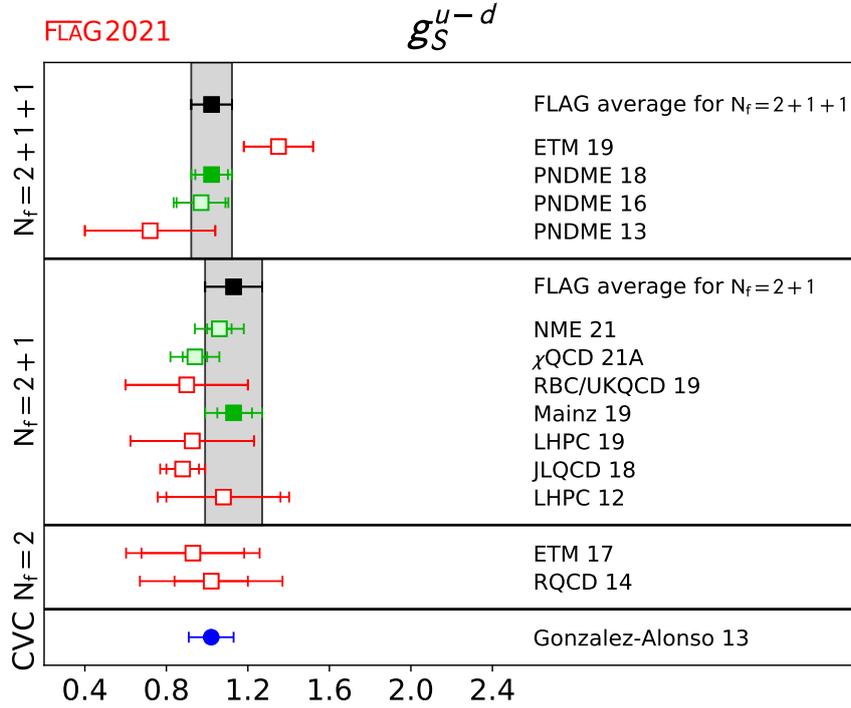


Figure 44: Lattice results and FLAG averages for the isovector scalar charge  $g_S^{u-d}$  for  $N_f = 2$ ,  $2+1$ , and  $2+1+1$  flavour calculations. Also shown is a phenomenological result obtained using the conserved vector current (CVC) relation [143] (circle).

The final FLAG value for  $g_S^{u-d}$  is

$$N_f = 2+1+1 : \quad g_S^{u-d} = 1.02(10) \quad \text{Ref. [50]}, \quad (435)$$

$$N_f = 2+1 : \quad g_S^{u-d} = 1.13(14) \quad \text{Ref. [84]}. \quad (436)$$

### 10.3.3 Results for $g_T^{u-d}$

Estimates of the isovector tensor charge are currently the most precise of the isovector charges with values that are stable over time, as can be seen from the compilation given in Tab. 71 and plotted in Fig. 45. This is a consequence of the smaller statistical fluctuations in the raw data and the very mild dependence on  $a$ ,  $M_\pi$ , and the lattice size  $M_\pi L$ . As a result, the uncertainty due to the various extrapolations is small. Also shown for comparison in Fig. 45 are phenomenological results using measures of transversity [169–173].

As in FLAG 19, for  $2+1+1$  flavours, only PNDME 18 [50], which supersedes PNDME 16 [46], PNDME 15 [44] and PNDME 13 [43], meets all the criteria for inclusion in the average. The details for this calculation are the same as those for  $g_S^{u-d}$  described in the previous section (Sec. 10.3.2), except that three-state fits were used to remove excited-state effects. The details of the  $2+1+1$  flavour calculation by ETM 19, which does not meet the criteria for averaging, are also the same as those described in the previous section for  $g_S^{u-d}$ .

For  $2+1$ -flavour calculations, only Mainz 19 [84] meets all criteria for inclusion in the averages. Details of this calculation are the same as for  $g_S^{u-d}$ , described in the previous section.

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$g_T^{u-d}$
ETM 19	[150]	2+1+1	A	■	○	★	★	○	0.936(25)
PNDME 18	[50]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	0.989(32)(10)
PNDME 16	[46]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	0.987(51)(20)
PNDME 15	[44, 45]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	1.020(76)
PNDME 13	[43]	2+1+1	A	■ <sup>‡</sup>	■	★	★	○	1.047(61)
NME 21	[151]	2+1	P	○ <sup>‡</sup>	★	★	★	○	0.95(5)(2)
RBC/UKQCD 19	[156]	2+1	A	■	○	★	★	■	1.04(5)
Mainz 19	[84]	2+1	A	★	○	★	★	○	0.965(38) <sub>(41)</sub> <sup>13</sup>
LHPC 19	[13]	2+1	A	■ <sup>‡</sup>	★	★	★	○	0.972(41)
JLQCD 18	[60]	2+1	A	■	○	○	★	○	1.08(3)(3)(9)
LHPC 12	[159]	2+1	A	■ <sup>‡</sup>	★	★	★	○	1.038(11)(12)
RBC/UKQCD 10D	[54]	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	[42]	2	A	■	○	○	★	○	1.004(21)(2)(19)
ETM 15D	[38]	2	A	■	○	○	★	○	1.027(62)
RQCD 14	[34]	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	[154]	2	A	■	■	■	★	■	0.93(6)

<sup>‡</sup> The rating takes into account that the action is not fully  $O(a)$  improved by requiring an additional lattice spacing.

Table 71: Overview of results for  $g_T^{u-d}$ .

Details for the 2+1-flavour NME 21, RBC/UKQCD 19, LHPC 19, Mainz 18, JLQCD 18, and LHPC 12A, calculations are identical to those presented previously in Sec. 10.3.2. The earlier RBC/UKQCD 10 calculation was performed using domain-wall fermions on the Iwasaki gauge action, with two volumes and several pion masses. The lowest pion mass used was  $M_\pi \sim 330$  MeV and does not meet the criteria for chiral extrapolation. In addition, the single lattice spacing and single source-sink separation do not meet the criteria for continuum extrapolation and excited states.

Two-flavour calculations include RQCD 14, with details identical to those described in Sec. 10.3.2. There are two calculations, ETM 15D [38] and ETM 17 [42], which employed twisted-mass fermions on the Iwasaki gauge action. The earlier work utilized three ensembles, with three volumes and two pion masses down to the physical point. The more recent work used only the physical pion mass ensemble. Both works used only a single lattice spacing  $a \sim 0.09$  fm, and therefore do not meet the criteria for continuum extrapolation. The early

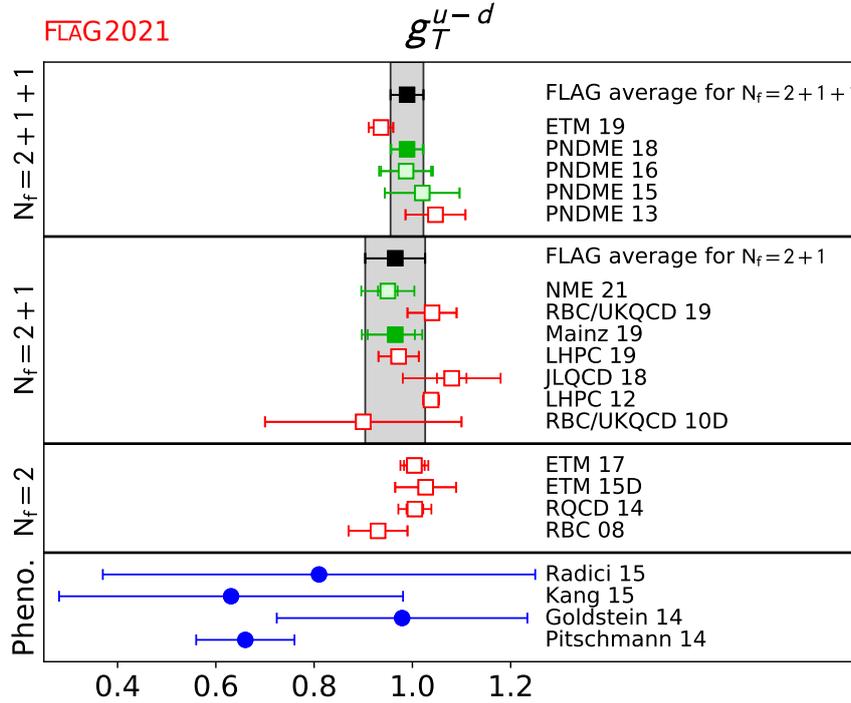


Figure 45: Lattice results and FLAG averages for the isovector tensor charge  $g_T^{u-d}$  for  $N_f = 2, 2+1,$  and  $2+1+1$  flavour calculations. Also shown are phenomenological results using measures of transversity [169–173] (circles).

work by RBC 08 with domain-wall fermions used three heavy values for the pion mass, and a single value for the lattice spacing, volume, and source-sink separation, and therefore do not meet many of the criteria.

The final FLAG value for  $g_T^{u-d}$  is

$$N_f = 2+1+1 : \quad g_T^{u-d} = 0.989(34) \quad \text{Ref. [50]}, \quad (437)$$

$$N_f = 2+1 : \quad g_T^{u-d} = 0.965(61) \quad \text{Ref. [84]}. \quad (438)$$

## 10.4 Flavour Diagonal Charges

Three examples of interactions for which matrix elements of flavour-diagonal operators ( $q\Gamma q$  where  $\Gamma$  defines the Lorentz structure of the bilinear quark operator) are needed are the neutral current interactions of neutrinos, elastic scattering of electrons off nuclei, and the scattering of dark matter off nuclei. In addition, these matrix elements also probe intrinsic properties of nucleons (the spin, the nucleon sigma term and strangeness content, and the contribution of the electric dipole moment (EDM) of the quarks to the nucleon EDM) as explained below. For brevity, all operators are assumed to be appropriately renormalized as discussed in Sec. 10.1.3.

The matrix elements of the scalar operator  $\bar{q}q$  with flavour  $q$  give the rate of change in the nucleon mass due to nonzero values of the corresponding quark mass. This relationship is given by the Feynman-Hellmann theorem. The quantities of interest are the nucleon  $\sigma$ -term,

$\sigma_{\pi N}$ , and the strange and charm content of the nucleon,  $\sigma_s$  and  $\sigma_c$ ,

$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle, \quad (439)$$

$$\sigma_s = m_s \langle N | \bar{s}s | N \rangle, \quad (440)$$

$$\sigma_c = m_c \langle N | \bar{c}c | N \rangle. \quad (441)$$

Here  $m_{ud}$  is the average of the up and down quark masses and  $m_s$  ( $m_c$ ) is the strange (charm) quark mass. The  $\sigma_{\pi N, s, c}$  give the shift in  $M_N$  due to nonzero light-, strange- and charm-quark masses. The same matrix elements are also needed to quantify the spin independent interaction of dark matter with nucleons. Note that, while  $\sigma_b$  and  $\sigma_t$  are also phenomenologically interesting, they are unlikely to be calculated on the lattice due to the expected tiny signal in the matrix elements. In principle, the heavy sigma terms can be estimated using  $\sigma_{u, d, s}$  by exploiting the heavy-quark limit [174–176].

The matrix elements of the axial operator  $\bar{q}\gamma_\mu\gamma_5q$  give the contribution  $\Delta q$  of quarks of flavour  $q$  to the spin of the nucleon:

$$\begin{aligned} \langle N | \bar{q}\gamma_\mu\gamma_5q | N \rangle &= g_A^q \bar{u}_N \gamma_\mu \gamma_5 u_N, \\ g_A^q \equiv \Delta q &= \int_0^1 dx (\Delta q(x) + \Delta \bar{q}(x)). \end{aligned} \quad (442)$$

The charge  $g_A^q$  is thus the contribution of the spin of a quark of flavour  $q$  to the spin of the nucleon. It is also related to the first Mellin moment of the polarized parton distribution function (PDF)  $\Delta q$  as shown in the second line in Eq. (442). Measurements by the European Muon collaboration in 1987 of the spin asymmetry in polarized deep inelastic scattering showed that the sum of the spins of the quarks contributes less than half of the total spin of the proton [177]. To understand this unexpected result, called the ‘‘proton spin crisis’’, it is common to start with Ji’s sum rule [178], which provides a gauge invariant decomposition of the nucleon’s total spin, as

$$\frac{1}{2} = \sum_{q=u, d, s, c} \left( \frac{1}{2} \Delta q + L_q \right) + J_g, \quad (443)$$

where  $\Delta q/2 \equiv g_A^q/2$  is the contribution of the intrinsic spin of a quark with flavour  $q$ ;  $L_q$  is the orbital angular momentum of that quark; and  $J_g$  is the total angular momentum of the gluons. Thus, to obtain the spin of the proton starting from QCD requires calculating the contributions of the three terms: the spin and orbital angular momentum of the quarks, and the angular momentum of the gluons. Lattice-QCD calculations of the various matrix elements needed to extract the three contributions are underway. An alternate decomposition of the spin of the proton has been provided by Jaffe and Manohar [179]. The two formulations differ in the decomposition of the contributions of the quark orbital angular momentum and of the gluons. The contribution of the quark spin, which is the subject of this review and given in Eq. (442), is the same in both formulations.

The tensor charges are defined as the matrix elements of the tensor operator  $\bar{q}\sigma^{\mu\nu}q$  with  $\sigma^{\mu\nu} = \{\gamma_\mu, \gamma_\nu\}/2$ :

$$g_T^q \bar{u}_N \sigma_{\mu\nu} u_N = \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle. \quad (444)$$

These flavour-diagonal tensor charges  $g_T^{u, d, s, c}$  quantify the contributions of the  $u$ ,  $d$ ,  $s$ ,  $c$  quark EDM to the neutron electric dipole moment (nEDM) [44, 180]. Since particles can have an

EDM only due to P and T (or CP assuming CPT is a good symmetry) violating interactions, the nEDM is a very sensitive probe of new sources of CP violation that arise in most extensions of the SM designed to explain nature at the TeV scale. The current experimental bound on the nEDM is  $d_n < 2.9 \times 10^{-26} e \text{ cm}$  [181], while the known CP violation in the SM implies  $d_n < 10^{-31} e \text{ cm}$  [182]. A nonzero result over the intervening five orders of magnitude would signal new physics. Planned experiments aim to reduce the bound to around  $10^{-28} e \text{ cm}$ . A discovery or reduction in the bound from these experiments will put stringent constraints on many BSM theories, provided the matrix elements of novel CP-violating interactions, of which the quark EDM is one, are calculated with the required precision.

One can also extract these tensor charges from the zeroth moment of the transversity distributions that are measured in many experiments including Drell-Yan and semi-inclusive deep inelastic scattering (SIDIS). Of particular importance is the active program at Jefferson Lab (JLab) to measure them [145, 146]. Transversity distributions describe the net transverse polarization of quarks in a transversely polarized nucleon. Their extraction from the data taken over a limited range of  $Q^2$  and Bjorken  $x$  is, however, not straightforward and requires additional phenomenological modeling. At present, lattice-QCD estimates of  $g_T^{u,d,s}$  are the most accurate [44, 147, 148] as can be deduced from Fig. 45. Future experiments will significantly improve the extraction of the transversity distributions. Thus, accurate calculations of the tensor charges using lattice QCD will continue to help elucidate the structure of the nucleon in terms of quarks and gluons and provide a benchmark against which phenomenological estimates utilizing measurements at JLab and other experimental facilities worldwide can be compared.

The methodology for the calculation of flavour-diagonal charges is also well-established. The major challenges are the much larger statistical errors in the disconnected contributions for the same computational cost and the need for the additional calculations of the isosinglet renormalization factors.

#### 10.4.1 Results for $g_A^{u,d,s}$

A compilation of results for the flavour-diagonal axial charges for the proton is given in Tab. 72 and plotted in Fig. 46. Results for the neutron can be obtained by interchanging the  $u$  and  $d$  flavour indices. Only two calculations already discussed in FLAG 19 [61] qualify for global averages: the PNDME 18A [49] for 2+1+1 flavours and the  $\chi$ QCD 18 [26] for 2+1 flavours.

The PNDME 18A [49] results were obtained using the 2+1+1 flavour clover-on-HISQ formulation. The connected contributions were obtained on 11 HISQ ensembles generated by the MILC collaboration with  $a \approx 0.057, 0.087, 0.12$  and  $0.15$  fm,  $M_\pi \approx 135, 220$  and  $320$  MeV, and  $3.3 < M_\pi L < 5.5$ . The light disconnected contributions were obtained on six of these ensembles with the lowest pion mass  $M_\pi \approx 220$  MeV, while the strange disconnected contributions were obtained on seven ensembles, i.e., including an additional one at  $a \approx 0.087$  fm and  $M_\pi \approx 135$  MeV. The excited state and the chiral-continuum fits were done separately for the connected and disconnected contributions, which introduces a systematic that is hypothesised to be small as explained in Ref. [49]. The analysis of the excited-state contamination, discussed in Sec. 10.1.2, was done using three-state fits for the connected contribution and two-state fits for the disconnected contributions. The chiral-continuum extrapolation was done keeping the leading correction terms proportional to  $M_\pi^2$  and  $a$  in both cases, and the leading finite-volume correction in  $M_\pi L$  was included in the analysis of the connected contributions. Isovector renormalization constants, calculated on the lattice in

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$\Delta u$	$\Delta d$
PNDME 20	[183]	2+1+1	C	★ <sup>‡</sup>	★	★	★	○	0.790(23)(30)	-0.425(15)(30)
ETM 19	[150]	2+1+1	A	■	○	★	★	○	0.862(17)	-0.424(16)
PNDME 18A	[49]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	0.777(25)(30) <sup>#</sup>	-0.438(18)(30) <sup>#</sup>
Mainz 19A	[184]	2+1	C	★	○	★	★	○	0.84(3)(4)	-0.40(3)(4)
$\chi$ QCD 18	[26]	2+1	A	○	★	★	★	○	0.847(18)(32) <sup>§</sup>	-0.407(16)(18) <sup>§</sup>
ETM 17C	[41]	2	A	■	○	○	★	○	0.830(26)(4)	-0.386(16)(6)
$\Delta s$										
PNDME 20	[183]	2+1+1	C	★ <sup>‡</sup>	★	★	★	○	-0.053(7)	
ETM 19	[150]	2+1+1	A	■	○	★	★	○	-0.0458(73)	
PNDME 18A	[49]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	-0.053(8) <sup>#</sup>	
Mainz 19A	[184]	2+1	C	★	○	★	★	○	-0.044(4)(5)	
$\chi$ QCD 18	[26]	2+1	A	○	★	★	★	○	-0.035(6)(7) <sup>§</sup>	
JLQCD 18	[60]	2+1	A	■	○	○	★	○	-0.046(26)(9) <sup>#</sup>	
$\chi$ QCD 15	[57]	2+1	A	■	○	■	★	○	-0.0403(44)(78) <sup>#</sup>	
Engelhardt 12	[185]	2+1	A	■	○	■	★	○	-0.031(17) <sup>#</sup>	
ETM 17C	[41]	2	A	■	○	○	★	○	-0.042(10)(2)	

<sup>#</sup> Assumed that  $Z_A^{n.s.} = Z_A^s$ .

<sup>‡</sup> The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.

<sup>§</sup> For this partially quenched analysis the criteria are applied to the unitary points.

Table 72: Overview of results for  $g_A^q$ .

the RI-SMOM scheme and converted to  $\overline{\text{MS}}$ , are used for all three flavour diagonal operators.

The PNDME 20 [183] provided a status update to PNDME 18A [49] and presented results showing that flavour mixing in the calculation of renormalization constants is small, and the isovector renormalization factor is a good approximation for renormalizing flavour diagonal axial charges as discussed in Sec. 10.1.3. It is not considered for the average as it is a conference

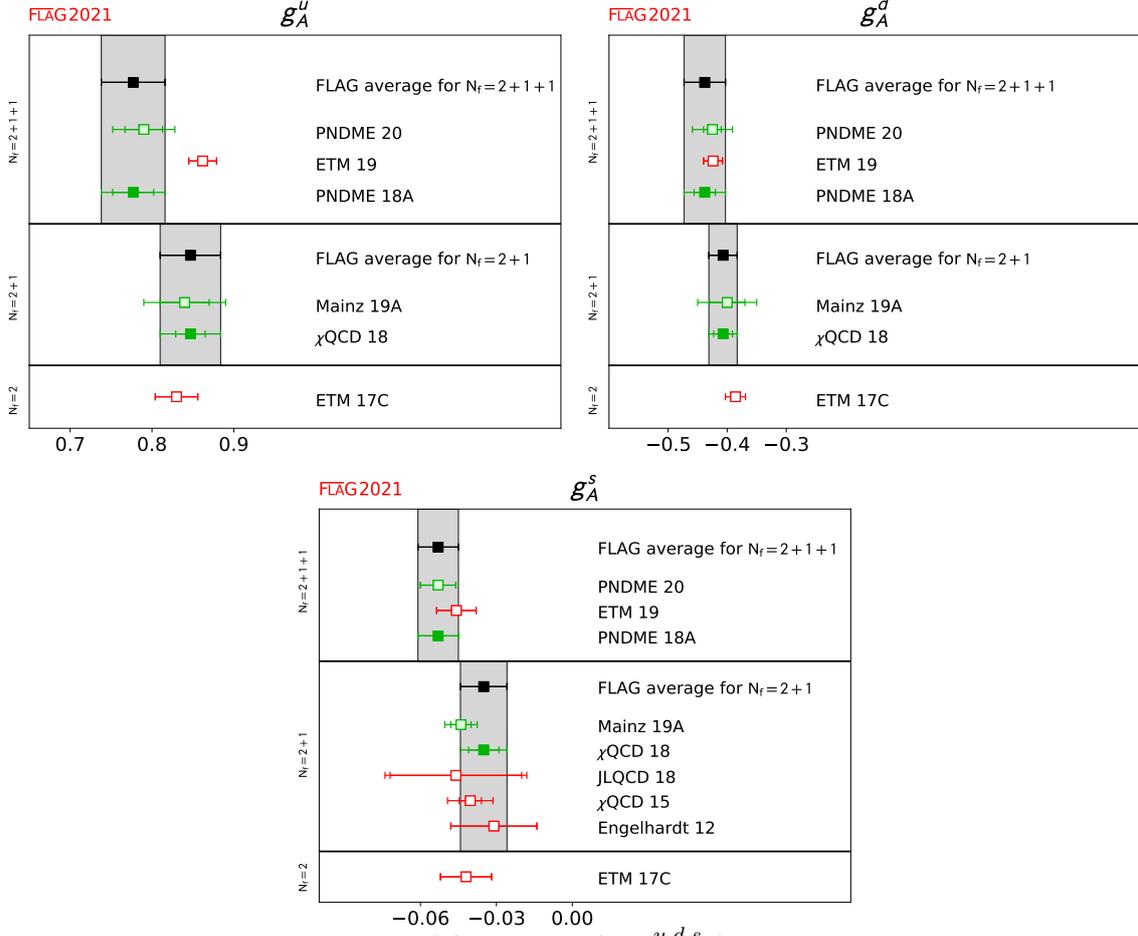


Figure 46: Lattice results and FLAG averages for  $g_A^{u,d,s}$  for the  $N_f = 2, 2+1,$  and  $2+1+1$  flavour calculations.

proceeding.

The ETM 19 [150] presented new results for  $g_A^{u,d,s,c}$  from a single ensemble with  $2+1+1$ -flavour twisted-mass fermions with a clover term at  $a = 0.0801(4)$  fm and  $M_\pi = 139.3(7)$  MeV. These are not considered for the averages as they do not satisfy the criteria for the continuum extrapolation.

The  $2+1+1$  flavour FLAG values for the axial charges  $g_A^{u,d,s}$  of the proton are, therefore, the same as the corresponding results given in Tab. 72 and unchanged from FLAG 19 [61]:

$$N_f = 2+1+1 : \quad g_A^u = 0.777(25)(30) \quad \text{Ref. [49],} \quad (445)$$

$$N_f = 2+1+1 : \quad g_A^d = -0.438(18)(30) \quad \text{Ref. [49],} \quad (446)$$

$$N_f = 2+1+1 : \quad g_A^s = -0.053(8) \quad \text{Ref. [49].} \quad (447)$$

There are also new results for  $g_A^{u,d,s}$  from Mainz 19A [184] with  $2+1$ -flavour ensembles. While they satisfy all the criteria, they are not included in the averages as [184] is a conference proceeding.

The  $2+1$  flavour FLAG results from  $\chi$ QCD 18 [26] were obtained using the overlap-on-domain-wall formalism. Three domain-wall ensembles with lattice spacings 0.143, 0.11

and 0.083 fm and sea-quark pion masses  $M_\pi = 171, 337$  and 302 MeV, respectively, were analyzed. In addition to the three approximately unitary points, the paper presents data for an additional 4–5 valence quark masses on each ensemble, i.e., partially quenched data. Separate excited-state fits were done for the connected and disconnected contributions. The continuum, chiral and volume extrapolation to the combined unitary and nonunitary data is made including terms proportional to both  $M_{\pi,\text{valence}}^2$  and  $M_{\pi,\text{sea}}^2$ , and two  $\mathcal{O}(a^2)$  discretization terms for the two different domain-wall actions. With just three unitary points, not all the coefficients are well constrained. The  $M_{\pi,\text{sea}}$  dependence is omitted and considered as a systematic, and a prior is used for the coefficients of the  $a^2$  terms to stabilize the fit. These  $\chi\text{QCD 18 2+1}$  flavour results for the proton, which supersede the  $\chi\text{QCD 15}$  [57] analysis, are

$$N_f = 2 + 1 : \quad g_A^u = 0.847(18)(32) \quad \text{Ref. [26]}, \quad (448)$$

$$N_f = 2 + 1 : \quad g_A^d = -0.407(16)(18) \quad \text{Ref. [26]}, \quad (449)$$

$$N_f = 2 + 1 : \quad g_A^s = -0.035(6)(7) \quad \text{Ref. [26]}. \quad (450)$$

The JLQCD 18 [60], ETM 17C [41] and Engelhardt 12 [185] calculations were not considered for the averages as they did not satisfy the criteria for the continuum extrapolation. All three calculations were done at a single lattice spacing. The JLQCD 18 calculation used overlap fermions and the Iwasaki gauge action. They perform a chiral fit using data at four pion masses in the range 290–540 MeV. Finite volume corrections are assumed to be negligible since each of the two pairs of points on different lattice volumes satisfy  $M_\pi L \geq 4$ . The ETM 17C calculation is based on a single twisted-mass ensemble with  $M_\pi = 130$  MeV,  $a = 0.094$  and a relatively small  $M_\pi L = 2.98$ . Engelhardt 12 [185] calculation was done on three asqtad ensembles with  $M_\pi = 293, 356$  and 495 MeV, but all at a single lattice spacing  $a = 0.124$  fm.

Results for  $g_A^s$  were also presented recently by LHPC in Ref. [5]. However, this calculation is not included in Tab. 72 as it has been performed on a single ensemble with  $a = 0.114$  fm and a heavy pion mass value of  $M_\pi \approx 317$  MeV.

#### 10.4.2 Results for $g_S^{u,d,s}$ from direct and hybrid calculations of the matrix elements

The sigma terms  $\sigma_q = m_q \langle N | \bar{q}q | N \rangle = m_q g_S^q$  or the quark mass fractions  $f_{T_q} = \sigma_q / M_N$  are normally computed rather than  $g_S^q$ . These combinations have the advantage of being renormalization group invariant in the continuum, and this holds on the lattice for actions with good chiral properties, see Sec. 10.1.3 for a discussion. In order to aid comparison with phenomenological estimates, e.g. from  $\pi$ - $N$  scattering [186–188], the light quark sigma terms are usually added to give the  $\pi N$  sigma term,  $\sigma_{\pi N} = \sigma_u + \sigma_d$ . The direct evaluation of the sigma terms involves the calculation of the corresponding three-point correlation functions for different source-sink separations  $\tau$ . For  $\sigma_{\pi N}$  there are both connected and disconnected contributions, while for most lattice fermion formulations only disconnected contributions are needed for  $\sigma_s$ . The techniques typically employed lead to the availability of a wider range of  $\tau$  for the disconnected contributions compared to the connected ones (both, however, suffer from signal-to-noise problems for large  $\tau$ , as discussed in Sec. 10.1) and we only comment on the range of  $\tau$  computed for the latter in the following.

Recent results for  $\sigma_{\pi N}$  and for  $\sigma_s$  from the direct approach are compiled in Tab. 73. ETM 19 [150] (discussed below) is the only new study included in this table since the last FLAG

report. Ref. [189] is also new, however, it was submitted to the arXiv after the deadline and will be reviewed in the next edition. For completeness, the descriptions of other works are reproduced from FLAG 19.

For both  $\sigma_{\pi N}$  and for  $\sigma_s$ , only the results from  $\chi$ QCD 15A [56] qualify for global averaging. In this mixed-action study, three RBC/UKQCD  $N_f = 2 + 1$  domain-wall ensembles are analyzed comprising two lattice spacings,  $a = 0.08$  fm with  $M_{\pi, \text{sea}} = 300$  MeV and  $a = 0.11$  fm with  $M_{\pi, \text{sea}} = 330$  MeV and 139 MeV. Overlap fermions are employed with a number of nonunitary valence quark masses. The connected three-point functions are measured with three values of  $\tau$  in the range 0.9–1.4 fm. A combined chiral, continuum and volume extrapolation is performed for all data with  $M_\pi < 350$  MeV. The leading order expressions are taken for the lattice-spacing and volume dependence while partially quenched  $SU(2)$  HB $\chi$ PT up to  $M_\pi^3$  terms models the chiral behaviour for  $\sigma_{\pi N}$ . The strange-quark sigma term has a milder dependence on the pion mass and only the leading-order quadratic terms are included in this case.

The lack of other qualifying studies is an indication of the difficulty and computational expense of performing these calculations. Nonetheless, this situation is likely to improve in the future. We note that although the recent analyses, ETM 16A [39], the new study ETM 19 [150] and JLQCD 18 [60], are at a single lattice spacing ( $a = 0.09$  fm, 0.08 fm and 0.11 fm, respectively), they satisfy the criteria for chiral extrapolation, finite volume and excited states. ETM 16A is a single ensemble study with  $N_f = 2$  twisted-mass fermions with a pion mass close to the physical point and  $M_\pi L = 3.0$ . Excited states are investigated utilizing  $\tau = 0.9$  fm up to  $\tau = 1.7$  fm for the connected three-point functions. In ETM 19 a high statistics analysis was carried out employing a  $N_f = 2 + 1 + 1$  physical point ensemble and seven source-sink separations in the range  $\tau = 0.6$ –1.6 fm, improving the precision they obtain for both  $\sigma_{\pi N}$  and  $\sigma_s$  compared to their  $N_f = 2$  results. JLQCD in JLQCD 18 utilize  $N_f = 2 + 1$  overlap fermion ensembles with pion masses reaching down to 293 MeV ( $M_\pi L = 4.0$ ) and apply techniques which give a wide range of  $\tau$  for the connected contribution, with the final results extracted from  $\tau \geq 1.2$  fm.

RQCD (RQCD 16 [35]) investigates the continuum, physical quark mass and infinite-volume limits, where the lattice spacing spans the range 0.06–0.08 fm, the minimum  $M_\pi$  is 150 MeV and  $M_\pi L$  is varied between 3.4 to 6.7 at  $M_\pi = 290$  MeV. This  $N_f = 2$  study has a red tag for the excited state criterion as multiple source-sink separations for the connected three-point functions are only computed on a subset of the ensembles. Clover fermions are employed and the lack of good chiral properties for this action means that there is mixing between quark flavours under renormalization when determining  $\sigma_s$  and a gluonic term needs to be considered for full  $\mathcal{O}(a)$  improvement (which has not been included, see Sec. 10.1.3 for a discussion).

Earlier work focuses only on  $\sigma_s$ . The analysis of JLQCD 12A [59], is performed on the same set of ensembles as the JLQCD 18 study discussed above and in addition includes smaller volumes for the lightest two pion masses.<sup>6</sup> No significant finite-volume effects are observed. Engelhardt 12 [185] and  $\chi$ QCD 13A [55] have less control over the systematics. The former is a single lattice spacing analysis restricted to small spatial volumes while the latter is a partially quenched study on a single ensemble with unitary  $M_\pi > 300$  MeV.

MILC has also computed  $\sigma_s$  using a hybrid method [191] which makes use of the Feynman-

<sup>6</sup>JLQCD also determine  $f_{T_s}$  in Ref. [192] in a single lattice spacing study on small volumes with heavy pion masses.

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$\sigma_{\pi N}$ [MeV]	$\sigma_s$ [MeV]
ETM 19	[150]	2+1+1	A	■	○	★	na/na	○	41.6(3.8)	45.6(6.2)
JLQCD 18	[60]	2+1	A	■	○	○	na/na	○	26(3)(5)(2)	17(18)(9)
$\chi$ QCD 15A	[56]	2+1	A	○	★	★	na/na	○	45.9(7.4)(2.8) <sup>§</sup>	40.2(11.7)(3.5) <sup>§</sup>
$\chi$ QCD 13A	[55]	2+1	A	■	■	○	-/na	○	-	33.3(6.2) <sup>§</sup>
JLQCD 12A	[59]	2+1	A	■	○	○	-/na	○	-	0.009(15)(16) $\times m_N^\dagger$
Engelhardt 12	[185]	2+1	A	■	○	■	-/na	○	-	0.046(11) $\times m_N^\dagger$
ETM 16A	[39]	2	A	■	○	○	na/na	○	37.2(2.6)( $\frac{4.7}{2.9}$ )	41.1(8.2)( $\frac{7.8}{5.8}$ )
RQCD 16	[35]	2	A	○	★	★	na/★	■	35(6)	35(12)
MILC 12C	[190]	2+1+1	A	★	★	★	-/○	○	-	0.44(8)(5) $\times m_s^{\P\S}$
MILC 12C	[190]	2+1	A	★	○	★	-/○	○	-	0.637(55)(74) $\times m_s^{\P\S}$
MILC 09D	[191]	2+1	A	★	○	★	-/na	○	-	59(6)(8) <sup>§</sup>

The renormalization criteria is given for  $\sigma_{\pi N}$  (first) and  $\sigma_s$  (second). The label 'na' indicates that no renormalization is required.

<sup>§</sup> For this partially quenched analysis the criteria are applied to the unitary points.

<sup>†</sup> This study computes the strange quark fraction  $f_{T_s}/m_N$ .

<sup>§</sup> This study employs a hybrid method, see Ref. [191].

<sup>¶</sup> The matrix element  $\langle N | \bar{s}s | N \rangle$  at the scale  $\mu = 2$  GeV in the  $\overline{\text{MS}}$  scheme is computed.

Table 73: Overview of results for  $\sigma_{\pi N}$  and  $\sigma_s$  from the direct approach (above) and  $\sigma_s$  from the hybrid approach (below).

Hellmann (FH) theorem and involves evaluating the nucleon matrix element  $\langle N | \int d^4x \bar{s}s | N \rangle$ .<sup>7</sup> This method is applied in MILC 09D [191] to the  $N_f = 2 + 1$  asqtad ensembles with lattice spacings  $a = 0.06, 0.09, 0.12$  fm and values of  $M_\pi$  ranging down to 224 MeV. A continuum and chiral extrapolation is performed including terms linear in the light-quark mass and quadratic in  $a$ . As the coefficient of the discretization term is poorly determined, a Bayesian prior is used, with a width corresponding to a 10% discretization effect between the continuum limit and the coarsest lattice spacing.<sup>8</sup> A similar updated analysis is presented in MILC 12C [190],

<sup>7</sup>Note that in the direct method the matrix element  $\langle N | \int d^3x \bar{s}s | N \rangle$ , involving the spatial volume sum, is evaluated for a fixed timeslice.

<sup>8</sup>This is consistent with discretization effects observed in other quantities at  $a = 0.12$  fm.

with an improved evaluation of  $\langle N | \int d^4x \bar{s}s | N \rangle$  on a subset of the  $N_f = 2 + 1$  asqtad ensembles. The study is also extended to HISQ  $N_f = 2 + 1 + 1$  ensembles comprising four lattice spacings with  $a = 0.06\text{--}0.15$  fm and a minimum pion mass of 131 MeV. Results are presented for  $g_S^s = \langle N | \bar{s}s | N \rangle$  (in the  $\overline{\text{MS}}$  scheme at 2 GeV) rather than for  $\sigma_s$ . The scalar matrix element is renormalized for both three and four flavours using the 2-loop factor for the asqtad action [193]. The error incurred by applying the same factor to the HISQ results is expected to be small.<sup>9</sup>

Both MILC 09D and MILC 12C achieve green tags for all the criteria, see Tab. 73. As the same set of asqtad ensembles is utilized in both studies we take MILC 12C as superseding MILC 09D for the three-flavour case. The global averaging is discussed in Sec. 10.4.4.

### 10.4.3 Results for $g_S^{u,d,s}$ using the Feynman-Hellmann theorem

An alternative approach for accessing the sigma terms is to determine the slope of the nucleon mass as a function of the quark masses, or equivalently, the squared pseudoscalar meson masses. The Feynman-Hellman (FH) theorem gives

$$\sigma_{\pi N} = m_u \frac{\partial M_N}{\partial m_u} + m_d \frac{\partial M_N}{\partial m_d} \approx M_\pi^2 \frac{\partial M_N}{\partial M_\pi^2}, \quad \sigma_s = m_s \frac{\partial M_N}{\partial m_s} \approx M_{\bar{s}s}^2 \frac{\partial M_N}{\partial M_{\bar{s}s}^2}, \quad (451)$$

where the fictitious  $\bar{s}s$  meson has a mass squared  $M_{\bar{s}s}^2 = 2M_K^2 - M_\pi^2$ . In principle this is a straightforward method as the nucleon mass can be extracted from fits to two-point correlation functions, and a further fit to  $M_N$  as a function of  $M_\pi$  (and also  $M_K$  for  $\sigma_s$ ) provides the slope. Nonetheless, this approach presents its own challenges: a functional form for the chiral behaviour of the nucleon mass is needed, and while baryonic  $\chi$ PT (B $\chi$ PT) is the natural choice, the convergence properties of the different formulations are not well established. Results are sensitive to the formulation chosen and the order of the expansion employed. If there is an insufficient number of data points when implementing higher order terms, the coefficients are sometimes fixed using additional input, e.g. from analyses of experimental data. This may influence the slope extracted. Simulations with pion masses close to or bracketing the physical point can alleviate these difficulties. In some studies the nucleon mass is used to set the lattice spacing. This naturally forces the fit to reproduce the physical nucleon mass at the physical point and may affect the extracted slope. Note that, if the nucleon mass is fitted as a function of the pion and kaon masses, the dependence of the meson masses on the quark masses also, in principle, needs to be considered in order to extract the sigma terms.

An overview of recent determinations of  $\sigma_{\pi N}$  and  $\sigma_s$  is given in Tab. 74. BMW 20A [195] (discussed below) is the only new study since the last FLAG report. For completeness, the descriptions of other works are reproduced from FLAG 19. Note that the renormalization criterion is not included in Tab. 74 as renormalization is not normally required when computing the sigma terms in the Feynman-Hellmann approach.<sup>10</sup> At present, a rating indicating control over excited state contamination is also not considered since a wide range of source-sink separations are available for nucleon two-point functions and ground state dominance is normally achieved. This issue may be revisited in the future as statistical precision improves and this systematic is further investigated.

<sup>9</sup>At least at 1-loop the  $Z$  factors for HISQ and asqtad are very similar, cf. Ref. [194].

<sup>10</sup>An exception to this is when clover fermions are employed. In this case one must take care of the mixing between quark flavours when renormalizing the quark masses that appear in Eq. (451).

Collaboration	Ref.	$N_f$		publication status	continuum extrapolation	chiral extrapolation	finite volume	$\sigma_{\pi N}$ [MeV]	$\sigma_s$ [MeV]
BMW 20A	[195]	1+1+1+1	P	★ <sup>‡</sup>	★	★		$0.0398(32)(44) \times m_N^\dagger$	$0.0577(46)(33) \times m_N^\dagger$
ETM 14A	[196]	2+1+1	A	★	○	○		$64.9(1.5)(13.2)^\Delta$	–
BMW 15	[197]	2+1	A	★ <sup>‡</sup>	★	★		$38(3)(3)$	$105(41)(37)$
Junnarkar 13	[198]	2+1	A	○	○	○		–	$48(10)(15)$
Shanahan 12	[199]	2+1	A	■	○	○		$45(6)/51(7)^*$	$21(6)/59(6)^*$
JLQCD 12A	[59]	2+1	A	■	○	○		–	$0.023(29)(28) \times m_N^\dagger$
QCDSF 11	[200]	2+1	A	■	■	○		$31(3)(4)$	$71(34)(59)$
BMW 11A	[201]	2+1	A	○ <sup>‡</sup>	★	○		$39(4)_{(7)}^{18}$	$67(27)_{(47)}^{55}$
Martin Camalich 10	[202]	2+1	A	■	★	■		$59(2)(17)$	$-4(23)(25)$
PACS-CS 09	[37]	2+1	A	■	★	■		$75(15)$	–
Walker-Loud 08	[203]	2+1	A	■	○	■		$84(17)(20)/42(14)(9)^*$	–
QCDSF 12	[31]	2	A	○	★	○		$37(8)(6)$	–
JLQCD 08B	[58]	2	A	■	○	■		$53(2)_{(7)}^{21}$	–

<sup>Δ</sup> Two results for  $\sigma_{\pi N}$  are quoted arising from different fit ansätze to the nucleon mass. The systematic error is the same as in Ref. [204] for a combined  $N_f = 2$  and  $N_f = 2 + 1 + 1$  analysis [205].

<sup>‡</sup> The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.

\* Two results are quoted.

<sup>†</sup> The quark fractions  $f_{T_{ud}} = f_{T_u} + f_{T_d} = \sigma_{\pi N}/m_N$  and/or  $f_{T_s} = \sigma_s/m_N$  are computed.

Table 74: Overview of results for  $\sigma_{\pi N}$  and  $\sigma_s$  from the Feynman-Hellmann approach.

There are several results for  $\sigma_{\pi N}$  that can be included in a global average. For  $N_f = 2$ , one study meets the selection criteria.<sup>11</sup> The analysis of QCDSF 12 [31] employs nonperturbatively improved clover fermions over three lattice spacings ( $a = 0.06$ – $0.08$  fm) with pion masses reaching down to around 160 MeV. Finite volume corrected nucleon masses are extrapolated via  $\mathcal{O}(p^4)$  covariant B $\chi$ PT with three free parameters. The other coefficients are taken from experiment, phenomenology or FLAG, with the corresponding uncertainties accounted for in the fit for those coefficients that are not well known. The nucleon mass is used to set the scale. A novel feature of this study is that a direct determination of  $\sigma_{\pi N}$  at around  $M_\pi = 290$  MeV was used as an additional constraint on the slope.

Turning to  $N_f = 2+1$ , two studies performed by the BMW collaboration and one by  $\chi$ QCD

<sup>11</sup>The ETM collaboration also determine  $\sigma_{\pi N}$  in Ref. [206] as part of an  $N_f = 2$  analysis to determine the lattice spacing from the nucleon mass. However, no final result is given.

are relevant. In BMW 11A [201], stout-smear tree-level clover fermions are employed on 15 ensembles with simulation parameters encompassing  $a = 0.06\text{--}0.12$  fm,  $M_\pi \sim 190\text{--}550$  MeV and  $M_\pi L \gtrsim 4$ . Taylor, Padé and covariant  $SU(3)$  B $\chi$ PT fit forms are considered. Due to the use of smeared gauge links, discretization effects are found to be mild even though the fermion action is not fully  $\mathcal{O}(a)$  improved. Fits are performed including an  $\mathcal{O}(a)$  or  $\mathcal{O}(a^2)$  term and also without a lattice-spacing dependent term. Finite volume effects were assessed to be small in an earlier work [207]. The final results are computed considering all combinations of the fit ansatz weighted by the quality of the fit. In BMW 15 [197], a more extensive analysis on 47 ensembles is presented for HEX-smear clover fermions involving five lattice spacings and pion masses reaching down to 120 MeV. Bracketing the physical point reduces the reliance on a chiral extrapolation. Joint continuum, chiral and infinite-volume extrapolations are carried out for a number of fit parameterisations with the final results determined via the Akaike information criterion procedure [132]. Although only  $\sigma_{\pi N}$  is accessible in the FH approach in the isospin limit, the individual quark fractions  $f_{T_q} = \sigma_q/M_N$  for  $q = u, d$  for the proton and the neutron are also quoted in BMW 15, using isospin relations.<sup>12</sup>

Regarding  $N_f = 2 + 1 + 1$ , there is only one recent study. In ETM 14A [196], fits are performed to the nucleon mass utilizing  $SU(2)$   $\chi$ PT for data with  $M_\pi \geq 213$  MeV as part of an analysis to set the lattice spacing. The expansion is considered to  $\mathcal{O}(p^3)$  and  $\mathcal{O}(p^4)$ , with two and three of the coefficients as free parameters, respectively. The difference between the two fits is taken as the systematic error. No discernable discretization or finite-volume effects are observed where the lattice spacing is varied over the range  $a = 0.06\text{--}0.09$  fm and the spatial volumes cover  $M_\pi L = 3.4$  up to  $M_\pi L > 5$ . The results are unchanged when a near physical point  $N_f = 2$  ensemble is added to the analysis in Ref. [204].

Since FLAG 19, BMW have performed a new  $N_f = 1 + 1 + 1 + 1$  study BMW 20A [195]. A two step analysis is followed: the dependence of the nucleon mass on the pion and kaon masses is determined on HEX-smear clover ensembles with  $a = 0.06\text{--}0.1$  fm and pion masses in the range  $M_\pi = 195\text{--}420$  MeV. The meson masses as a function of the quark masses are evaluated on stout-staggered ensembles with a similar range in  $a$  and quark masses which bracket their physical values. As [195] is a preprint, the result is not considered for the average.

We note that the  $N_f = 2 + 1$  study by  $\chi$ QCD [209] based on overlap valence fermions on four domain-wall fermion ensembles with  $a = 0.08\text{--}0.14$  fm and  $M_\pi$  down to the physical point is also new. However, since  $\sigma_{\pi N}$  is determined from a single fit and the systematic uncertainties are not estimated, we do not present the result in the table.

Other determinations of  $\sigma_{\pi N}$  in Tab. 74 receive one or more red tags. Walker-Loud 08 [203], JLQCD 08B [58], PACS-CS 09 [37] and QCDSF 11 [200] are single lattice spacing studies. In addition, the volume for the minimum pion mass is rather small for Walker-Loud 08, JLQCD 08B and PACS-CS 09, while QCDSF 11 is restricted to heavier pion masses.

We also consider publications that are based on results for baryon masses found in the literature. As different lattice setups (in terms of  $N_f$ , lattice actions, etc.) will lead to different systematics, we only include works in Tab. 74 which utilize a single setup. These correspond to Shanahan 12 [199] and Martin Camalich 10 [202], which fit PACS-CS data [210] (the PACS-CS 09 study is also based on these results). Note that Shanahan 12 avoids a red tag for the volume criterion as the lightest pion mass ensemble is omitted. Recent studies which combine data from different setups/collaborations are displayed for comparison in Figs. 47 and 48 in the next section.

<sup>12</sup>These isospin relations were also derived in Ref. [208].

Several of the above studies have also determined the strange quark sigma term. This quantity is difficult to access via the Feynman-Hellmann method since in most simulations the physical point is approached by varying the light-quark mass, keeping  $m_s$  approximately constant. While additional ensembles can be generated, it is hard to resolve a small slope with respect to  $m_s$ . Such problems are illustrated by the large uncertainties in the results from BMW 11A and BMW 15. Alternative approaches have been pursued in QCDSF 11, where the physical point is approached along a trajectory keeping the average of the light- and strange-quark masses fixed, and JLQCD 12A [59], where quark mass reweighting is applied. The latter is a single lattice spacing study. One can also fit to the whole baryon octet and apply  $SU(3)$  flavour symmetry constraints as investigated in, e.g. Martin Camalich 10, Shanahan 12, QCDSF 11 and BMW 11A.

The determinations of  $\sigma_s$  in BMW 11A and BMW 15 qualify for averaging. The mixed action study of Junnarkar 13 [198] with domain-wall valence fermions on MILC  $N_f = 2 + 1$  asqtad ensembles also passes the FLAG criteria. The derivative  $\partial M_N / \partial m_s$  is determined from simulations above and below the physical strange quark mass for  $M_\pi$  around 240–675 MeV. The resulting values of  $\sigma_s$  are extrapolated quadratically in  $M_\pi$ . The quark fraction  $f_{T_s} = \sigma_s / M_N$  exhibits a milder pion-mass dependence and extrapolations of this quantity were also performed using ansätze linear and quadratic in  $M_\pi$ . A weighted average of all three fits was used to form the final result. Two lattice spacings were analyzed, with  $a$  around 0.09 fm and 0.12 fm, however, discretization effects could not be resolved. We note that BMW in their  $N_f = 1 + 1 + 1 + 1$  study [195] significantly improve the precision of their estimate of  $\sigma_s$ . Even though all the criteria are satisfied, it is not considered for the average as Ref. [195] is a preprint. The global averaging of all calculations that qualify is discussed in the next section.

#### 10.4.4 Summary of Results for $g_S^{u,d,s}$

We consider computing global averages of results determined via the direct, hybrid and Feynman-Hellmann (FH) methods. These are unchanged from FLAG 19. Beginning with  $\sigma_{\pi N}$ , Tabs. 73 and 74 show that for  $N_f = 2 + 1 + 1$  only ETM 14A (FH) satisfies the selection criteria. We take this value as our FLAG result for the four-flavour case.

$$N_f = 2 + 1 + 1 : \quad \sigma_{\pi N} = 64.9(1.5)(13.2) \text{ MeV} \quad \text{Ref. [196]}. \quad (452)$$

We remark that although the  $N_f = 1 + 1 + 1 + 1$  BMW 20A study also satisfies the criteria, as Ref. [195] is a preprint this work is not considered for averaging. For  $N_f = 2 + 1$  we form an average from the BMW 11A (FH), BMW 15 (FH) and  $\chi$ QCD 15A (direct) results, yielding

$$N_f = 2 + 1 : \quad \sigma_{\pi N} = 39.7(3.6) \text{ MeV} \quad \text{Refs. [56, 197, 201]}. \quad (453)$$

Note that both BMW results are included as they were obtained on independent sets of ensembles (employing different fermion actions). The average is dominated by the BMW 15 calculation, which has much smaller overall errors compared to the other two studies.

Turning to the results for  $N_f = 2$ , only QCDSF 12 (FH) qualifies. This is taken as the FLAG result

$$N_f = 2 : \quad \sigma_{\pi N} = 37(8)(6) \text{ MeV} \quad \text{Ref. [31]}. \quad (454)$$

Moving on to  $\sigma_s$  and the calculations detailed in Tab. 73, for  $N_f = 2 + 1 + 1$  MILC 12C (hybrid) and BMW 20A satisfy the quality criteria, however, the latter is a preprint and is not

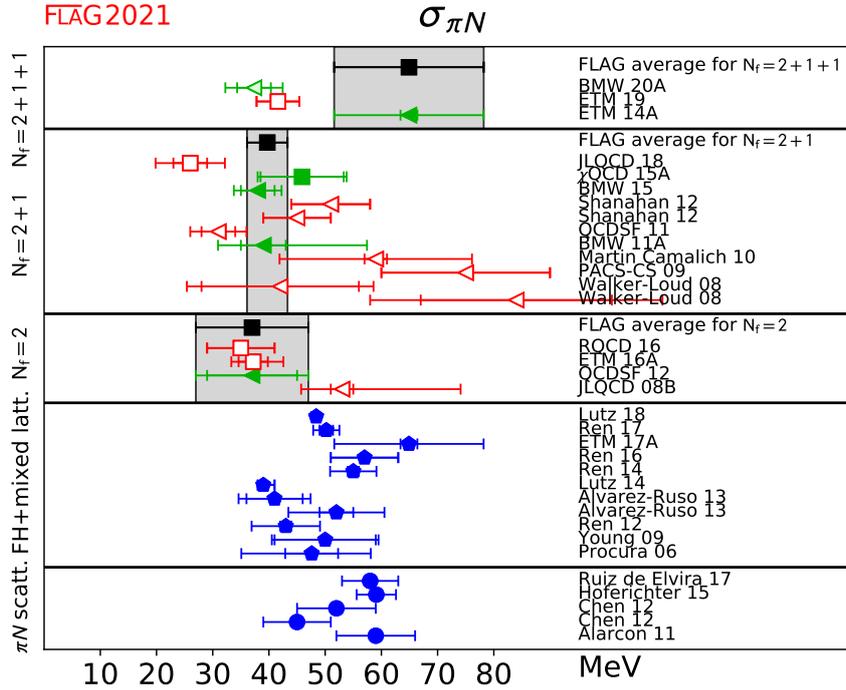


Figure 47: Lattice results and FLAG averages for the nucleon sigma term,  $\sigma_{\pi N}$ , for the  $N_f = 2, 2 + 1$ , and  $2 + 1 + 1$  flavour calculations. Determinations via the direct approach are indicated by squares and the Feynman-Hellmann method by triangles. Results from calculations which analyze more than one lattice data set within the Feynman-Hellmann approach [204, 211–219] are shown for comparison (pentagons) along with those from recent analyses of  $\pi$ - $N$  scattering [186–188, 220] (circles).

considered for averaging. In order to convert the result for  $\langle N | \bar{s}s | N \rangle$  given in MILC 12C to a value for  $\sigma_s$ , we multiply by the appropriate FLAG average for  $m_s$  given in Eq. (35) of FLAG 19. This gives our result for four flavours.

$$N_f = 2 + 1 + 1 : \quad \sigma_s = 41.0(8.8) \text{ MeV} \quad \text{Ref. [190].} \quad (455)$$

For  $N_f = 2 + 1$  we perform a weighted average of BMW 11A (FH), MILC 12C (hybrid), Junnarkar 13 (FH), BMW 15 (FH) and  $\chi$ QCD 15A (direct). MILC 09D [191] also passes the FLAG selection rules, however, this calculation is superseded by MILC 12C. As for Eq. (455), the strangeness scalar matrix element determined in the latter study is multiplied by the three flavour FLAG average for  $m_s$  given in Eq. (33) of FLAG 19. There are correlations between the MILC 12C and Junnarkar 13 results as there is some overlap between the sets of asqtad ensembles used in both cases. To be conservative we take the statistical errors for these two studies to be 100% correlated. The global average is

$$N_f = 2 + 1 : \quad \sigma_s = 52.9(7.0) \text{ MeV} \quad \text{Refs. [56, 190, 197, 198, 201].} \quad (456)$$

Given that all of the  $N_f = 2$  studies have at least one red tag we are not able to give an average in this case.

All the results for  $\sigma_{\pi N}$  and  $\sigma_s$  are displayed in Figs. 47 and 48 along with the averages given above. Note that where  $f_{T_{ud}} = f_{T_u} + f_{T_d}$  or  $f_{T_s}$  is quoted in Tabs. 73 and 74, we

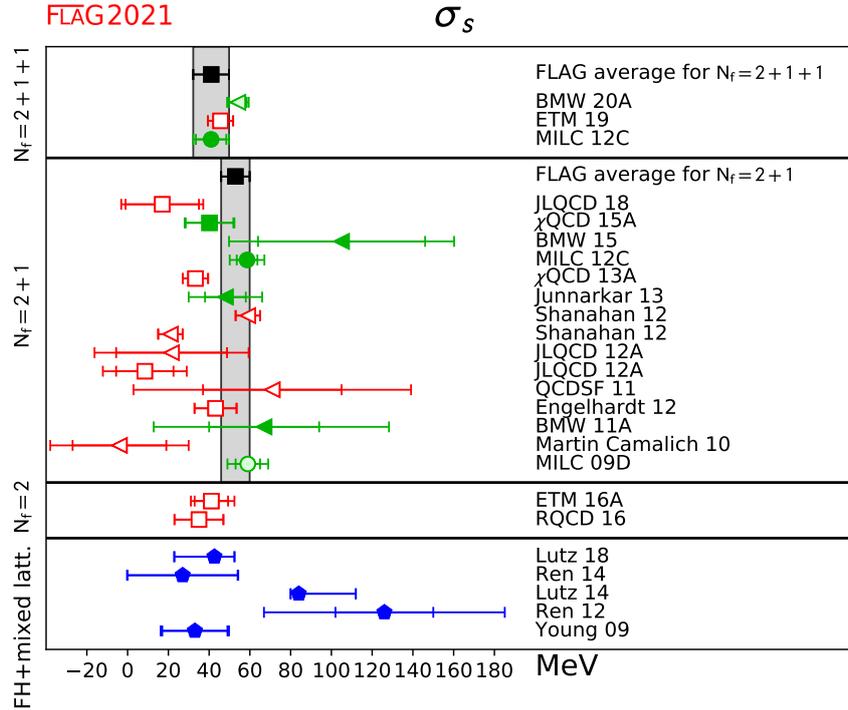


Figure 48: Lattice results and FLAG averages for  $\sigma_s$  for the  $N_f = 2, 2 + 1,$  and  $2 + 1 + 1$  flavour calculations. Determinations via the direct approach are indicated by squares, the Feynman-Hellmann method by triangles and the hybrid approach by circles. Results from calculations which analyze more than one lattice data set within the Feynman-Hellmann approach [212, 213, 215, 216, 219] are shown for comparison (pentagons).

multiply by the experimental proton mass in order to include the results in the figures. Those results which pass the FLAG criteria, shown in green, are reasonably consistent. However, there is some fluctuation in the central values, in particular, when taking the lattice results as a whole into account, and we caution the reader that the averages may change as new results become available.

Also shown for comparison in the figures are determinations from the FH method which utilize more than one lattice data set [204, 211–219] as well as results for  $\sigma_{\pi N}$  obtained from recent analyses of  $\pi$ - $N$  scattering [186–188, 220]. There is some tension, at the level of three to four standard deviations, between the lattice average for  $N_f = 2 + 1$  and Hoferichter et al. [188] (Hoferichter 15 in Fig. 47), who quote a precision similar to that of the average.

Finally we remark that, by exploiting the heavy-quark limit, the light- and strange-quark sigma terms can be used to estimate  $\sigma_q$  for the charm, bottom and top quarks [174–176]. The resulting estimate for the charm quark, see, e.g. the RQCD 16  $N_f = 2$  analysis of Ref. [35] that reports  $f_{T_c} = 0.075(4)$  or  $\sigma_c = 70(4)$  MeV is consistent with the direct determinations of ETM 19 [150] for  $N_f = 2 + 1 + 1$  of  $\sigma_c = 107(22)$  MeV, ETM 16A [39] for  $N_f = 2$  of  $\sigma_c = 79(21)_{(8)}^{(12)}$  MeV and  $\chi$ QCD 13A [55] for  $N_f = 2 + 1$  of  $\sigma_c = 94(31)$  MeV. BMW in BMW 20A [195] employing the Feynman-Hellmann approach obtain  $f_{T_c} = \sigma_c/m_N = 0.0734(45)(55)$  for  $N_f = 1 + 1 + 1 + 1$ . MILC in MILC 12C [190] find  $\langle N|\bar{c}c|N\rangle = 0.056(27)$  in the  $\overline{\text{MS}}$  scheme at a scale of 2 GeV for  $N_f = 2 + 1 + 1$  via the hybrid method. Considering the large uncertainty, this is consistent with the other results once multiplied by the charm quark mass.

### 10.4.5 Results for $g_T^{u,d,s}$

A compilation of recent results for the flavour-diagonal tensor charges  $g_T^{u,d,s}$  for the proton in the  $\overline{\text{MS}}$  scheme at 2 GeV is given in Tab. 75 and plotted in Fig. 49. Results for the neutron can be obtained by interchanging the  $u$  and  $d$  flavour indices. Only the PNDME 2+1+1 flavour calculations qualify for the global average.

The FLAG values remain the same as in FLAG 19, i.e., the PNDME 18B [48] results, which supersede the PNDME 16 [46] and the PNDME 15 [44] results:

$$N_f = 2 + 1 + 1 : \quad g_T^u = 0.784(28)(10) \quad \text{Ref. [48]}, \quad (457)$$

$$N_f = 2 + 1 + 1 : \quad g_T^d = -0.204(11)(10) \quad \text{Ref. [48]}, \quad (458)$$

$$N_f = 2 + 1 + 1 : \quad g_T^s = -0.0027(16) \quad \text{Ref. [48]}. \quad (459)$$

The ensembles and the analysis strategy used in PNDME 18B is the same as described in Sec. 10.4.1 for  $g_A^{u,d,s}$ . The only difference for the tensor charges was that a one-state (constant) fit was used for the disconnected contributions as the data did not show significant excited-state contamination. The isovector renormalization constant, used for all three flavour-diagonal tensor operators, was calculated on the lattice in the RI-SMOM scheme and converted to  $\overline{\text{MS}}$  at 2 GeV using 2-loop perturbation theory.

The PNDME 20 [183] provided a status update on  $g_T^{u,d,s}$  to PNDME 18B [49] but is not considered for the average as it is a conference proceeding. It also presented results showing that flavour mixing in the calculation of tensor renormalization constants is small, and the isovector renormalization factor is a good approximation for renormalizing flavour-diagonal tensor charges as discussed in Sec. 10.1.3.

The ETM 19 [150] presented new results for  $g_T^{u,d,s,c}$  from a single ensemble with 2+1+1-flavour twisted-mass fermions with a clover term at  $a = 0.0801(4)$  fm and  $M_\pi = 139.3(7)$  MeV. It was not considered for the final averages because it did not satisfy the criteria for the continuum extrapolation as already discussed in Sec. 10.4.1. The same applies to the JLQCD 18 [60] and ETM 17 [42] calculations. The Mainz 19A [184] results with 2+1-flavour ensembles of clover fermions are not included in the averages as Ref. [184] is a conference proceeding.

Collaboration	Ref.	$N_f$	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	$g_T^u$	$g_T^d$
PNDME 20	[183]	2+1+1	C	★ <sup>‡</sup>	★	★	★	○	0.783(27)(10)	−0.205(10)(10)
ETM 19	[150]	2+1+1	A	■	○	★	★	○	0.729(22)	−0.2075(75)
PNDME 18B	[48]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	0.784(28)(10) <sup>#</sup>	−0.204(11)(10) <sup>#</sup>
PNDME 16	[46]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	0.792(42) <sup>#&amp;</sup>	−0.194(14) <sup>#&amp;</sup>
PNDME 15	[44, 45]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	0.774(66) <sup>#</sup>	−0.233(28) <sup>#</sup>
Mainz 19A	[184]	2+1	C	★	○	★	★	○	0.77(4)(6)	−0.19(4)(6)
JLQCD 18	[60]	2+1	A	■	○	○	★	○	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	[42]	2	A	■	○	○	★	○	0.782(16)(2)(13)	−0.219(10)(2)(13)
$g_T^s$										
PNDME 20	[183]	2+1+1	C	★ <sup>‡</sup>	★	★	★	○	−0.0022(12)	
ETM 19	[150]	2+1+1	A	■	○	★	★	○	−0.00268(58)	
PNDME 18B	[48]	2+1+1	A	★ <sup>‡</sup>	★	★	★	○	−0.0027(16) <sup>#</sup>	
PNDME 15	[44, 45]	2+1+1	A	○ <sup>‡</sup>	★	★	★	○	0.008(9) <sup>#</sup>	
Mainz 19A	[184]	2+1	C	★	○	★	★	○	−0.0026(73)(42)	
JLQCD 18	[60]	2+1	A	■	○	○	★	○	−0.012(16)(8)	
ETM 17	[42]	2	A	■	○	○	★	○	−0.00319(69)(2)(22)	

<sup>‡</sup> The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.

<sup>#</sup> Assumed that  $Z_T^{n.s.} = Z_T^s$ .

<sup>&</sup> Disconnected terms omitted.

Table 75: Overview of results for  $g_T^q$ .

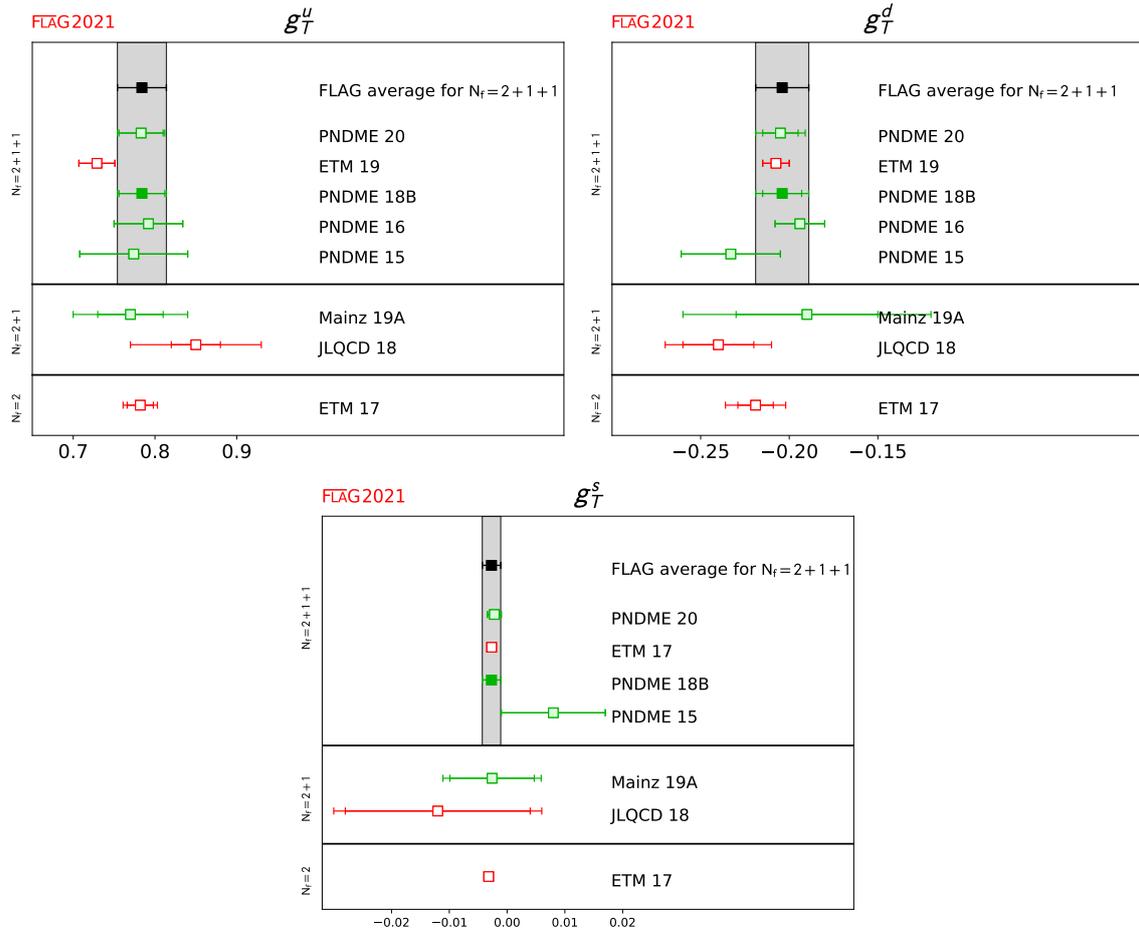


Figure 49: Lattice results and FLAG averages for  $g_T^{u,d,s}$  for the  $N_f = 2, 2 + 1,$  and  $2 + 1 + 1$  flavour calculations.

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