

C Notes

In the following Appendices we provide more detailed information on the simulations used to calculate the quantities discussed in Secs. 4–11. We present this information only for results that are new w.r.t. FLAG 21. For all other results the information is available in the corresponding Appendices C.1–C.9 in FLAG 21 [1], B.1–B.8 in FLAG 19 [2], and B.1–B.7 in FLAG 16 [3].

C.1 Notes to Sec. 4 on quark masses

Collab.	Ref.	N_f	a [fm]	Description
CLQCD 23	[4]	2+1	0.052, 0.077, 0.11	smeared Wilson-clover/Symanzik

Table 79: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud} , m_s and, in some cases m_u and m_d , with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
CLQCD 23	[4]	2+1	135.5	

Table 80: Chiral extrapolation/minimum pion mass in determinations of m_{ud} , m_s , and in some cases m_u and m_d , with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min} L$	Description
CLQCD 23	[4]	2+1	2.5 – 5.1	3.45	

Table 81: Finite-volume effects in determinations of m_{ud} , m_s and, in some cases m_u and m_d , with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
CLQCD 23	[4]	2+1	RI/MOM

Table 82: Renormalization in determinations of m_{ud} , m_s and, in some cases m_u and m_d , with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
ALPHA 23	[5]	2+1	0.085, 0.075, 0.064, 0.049	$\mathcal{O}(a^2)$ terms, with mass-dependent coefficients, are included in the chiral-continuum extrapolation. t_0 is used as intermediate scale with the physical scale set by a combination of f_π and f_K in the isosymmetric limit.

Table 83: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ALPHA 23	[5]	2+1	200	

Table 84: Chiral extrapolation/minimum pion mass in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ALPHA 23	[5]	2+1	2.7, 2.4, 3.1/4.1, 2.4/3.1	3.9, 5.1, 4.2, 4.1	No explicit discussion of FSE.

Table 85: Finite-volume effects in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
ALPHA 23	[5]	2+1	Schrödinger functional

Table 86: Renormalization in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

C.2 Notes to Sec. 5 on $|V_{ud}|$ and $|V_{us}|$

Collab.	Ref.	N_f	a [fm]	Description
PACS 22	[6]	2+1	0.085, 0.063	Nonperturbative $\mathcal{O}(a)$ clover quark action. Scale set from Ξ -baryon mass.

Table 87: Continuum extrapolations/estimation of lattice artifacts in the determinations of $f_+(0)$.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
PACS 22	[6]	2+1	135	Physical point simulation at a single pion mass 135 MeV.

Table 88: Chiral extrapolation/minimum pion mass in determinations of $f_+(0)$.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PACS 22	[6]	2+1	10.9	7.5	

Table 89: Finite-volume effects in determinations of $f_+(0)$.

Collab.	Ref.	N_f	a [fm]	Description
ETM 21	[7]	2+1+1	0.07, 0.08, 0.09	Wilson-clover twisted mass quark action. Relative scale through gradient flow scale w_0 and absolute scale through f_π .
CLQCD 23	[4]	2+1	0.105, 0.077, 0.052	Tadpole-improved clover quark action with one-step stout smearing. Scale set through gradient-flow scale w_0 .

Table 90: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π .

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 21	[7]	2+1+1	134	Chiral extrapolation based on NLO SU(2) χ PT.
CLQCD 23	[4]	2+1	135.5	Two additional partially-quenched quark masses on each ensemble. Chiral extrapolation based on NLO SU(2) partially-quenched χ PT for f_π and a polynomial extrapolation for f_K .

Table 91: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π .

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min} L$	Description
ETM 21	[7]	2+1+1	2.0–5.6	3.8	Three different volumes at $M_\pi = 253$ MeV and $a = 0.08$ fm.
CLQCD 23	[4]	2+1	2.5–5.1	3.5	Two volumes at four simulation points with $M_\pi = 200–300$ MeV on two coarser lattices.

Table 92: Finite-volume effects in determinations of f_K/f_π .

C.3 Notes to section 6 on Kaon mixing

C.3.1 Kaon B -parameter B_K

Collab.	Ref.	N_f	a [fm]	Description
RBC/UKQCD 24	[8]	2+1	0.114, 0.084, 0.073	Combined continuum and chiral (NLO. SU(2)) extrapolation fits. Assigned systematic error at the per-mille level

Table 93: Continuum extrapolations/estimation of lattice artifacts in determinations of B_K .

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RBC/UKQCD 24	[8]	2+1	139, 139, 232	Chiral extrapolations based on SU(2)- χ PT fits at NLO. Systematic uncertainties amount to less than half a per cent.

Table 94: Chiral extrapolation/minimum pion mass in determinations of B_K .

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
RBC/UKQCD 24	[8]	2+1	5.5, 5.4, 3.5, 2.6	3.9, 3.8, 4.1	Finite-volume effects are found to be negligible compared to other systematic effects and are thus omitted in the final error budget.

Table 95: Finite-volume effects in determinations of B_K .

Collab.	Ref.	N_f	Ren.	running match.	Description
RBC/UKQCD 24	[8]	2+1	RI	PT1 ℓ	Two different RI-SMOM schemes used to estimate a 1% systematic error owing to the perturbative matching to $\overline{\text{MS}}$.

Table 96: Running and matching in determinations of B_K .

C.3.2 Kaon BSM B -parameters

Collab.	Ref.	N_f	a [fm]	Description
RBC/UKQCD 24	[8]	2+1	0.114, 0.084, 0.073	Systematic uncertainties ranging from a minimum of 0.4% (for the case of B_2) to 1.9% (for the case of B_3).

Table 97: Continuum extrapolations/estimation of lattice artifacts in determinations of the BSM B_i parameters.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RBC/UKQCD 24	[8]	2+1	139, 139, 232	Chiral extrapolations based on SU(2)- χ PT fits at NLO. Systematic uncertainties amount to less than half a percent.

Table 98: Chiral extrapolation/minimum pion mass in determinations of the BSM B_i parameters.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
RBC/UKQCD 24	[8]	2+1	5.5, 5.4, 3.5 2.6	3.9, 3.8, 4.1	Finite-volume effects are at most at the 2 per-mille level. They are negligible compared to other systematic effects and are therefore omitted in the error budget.

Table 99: Finite-volume effects in determinations of the BSM B_i parameters.

Collab.	Ref.	N_f	Ren.	running match.	Description
RBC/UKQCD 24	[8]	2+1	RI	PT1 ℓ	Two different RI-SMOM schemes used to estimate the systematic error owing to the perturbative matching to $\overline{\text{MS}}$; minimal value of about 0.7% for the case of B_2 and maximal of 2.4% for B_3 .

Table 100: Running and matching in determinations of the BSM B_i parameters.

C.3.3 $K \rightarrow \pi\pi$ decay amplitudes

Collab.	Ref.	N_f	a [fm]	Description
RBC/UKQCD 23A	[9]	2+1	0.193	Single lattice spacing.

Table 101: Continuum extrapolations/estimation of lattice artifacts in determinations of the $K \rightarrow \pi\pi$ decay amplitudes.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RBC/UKQCD 23A	[9]	2+1	142.6	Single pion mass value, close to the physical point.

Table 102: Chiral extrapolation/minimum pion mass in determinations of the $K \rightarrow \pi\pi$ decay amplitudes.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min} L$	Description
RBC/UKQCD 23A	[9]	2+1	4.6	3.3	Finite-volume effects amount to a 7% systematic error contribution to the final error budget of A_0 and A_2 .

Table 103: Finite-volume effects in determinations of the $K \rightarrow \pi\pi$ decay amplitudes.

Collab.	Ref.	N_f	Ren.	running match.	Description
RBC/UKQCD 23A	[9]	2+1	RI	PT1 ℓ	Two different RI-SMOM schemes are used. One of the two schemes is used for the final analysis. A systematic error ranging from 6% to 16%, depending on the considered case, is included based on the dispersion of other sets of intermediate scheme and scales. Systematic uncertainties arising from the computation of the Wilson coefficients in the $\overline{\text{MS}}$ scheme amount to 12%.

Table 104: Running and matching in determinations of the $K \rightarrow \pi\pi$ decay amplitudes.

C.4 Notes to Sec. 7 on D -meson decay constants and form factors

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 13F ETM 14E ETM 21B	[10–12]	2+1+1	245, 239, 211 167, 137, 134	$f_{D_s}\sqrt{m_{D_s}}$ in ETM 13F and f_{D_s}/m_{D_s} in ETM 14E are extrapolated using both a quadratic and a linear fit in m_l plus $O(a^2)$ terms. In ETM 21B either w_0 or the $D_{(s)}$ meson mass are used as scaling variables in the chiral-continuum extrapolations.

Table 105: Chiral extrapolation/minimum pion mass in $N_f = 2 + 1 + 1$ determinations of the D - and D_s -meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RQCD/ALPHA 24	[13]	2+1	335, 129, 155, 130, 175, 337	The dependence on light and strange quark masses is described using fit ansätze inspired by NLO HM χ PT.
ALPHA 23	[5]	2+1	277, 415, 200, 257	HM χ PT expressions are used for the quantities $(8t_0)^{3/4}f_{D_{(s)}}\sqrt{m_{D_{(s)}}}$.

Table 106: Chiral extrapolation/minimum pion mass in $N_f = 2 + 1$ determinations of the D - and D_s -meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 13F ETM 14E ETM 21B	[10–12]	2+1+1	2.3/4.6, 2.6/5.2, 3.3/5.5	3.8, 3.6, 3.7	The comparison of two different volumes at the two largest lattice spacings indicates that FV effects are below the statistical errors. No explicit discussion of FSE in ETM 21B.

Table 107: Finite-volume effects in $N_f = 2 + 1 + 1$ determinations of the D - and D_s -meson decay constants. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
RQCD/ALPHA 24	[13]	2+1	2.34, 2.05/5.46, 2.4/4.8, 2.0/6.1, 2.4/4.8, 2.48	4, 3.6, 3.8, 4.05, 4.2, 4.2	By comparing different volumes (up to 5) at fixed pion mass, FSE are estimated to be negligible once the cut $m_\pi L \geq 3.5$ or $L \geq 2.3$ fm is imposed.
ALPHA 23	[5]	2+1	2.7, 2.4, 3.1/4.1, 2.4/3.1	3.9, 5.1, 4.2, 4.1	No explicit discussion of FSE.

Table 108: Finite-volume effects in $N_f = 2 + 1$ determinations of the D - and D_s -meson decay constants. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
ETM 13F ETM 14E ETM 21B	[10–12]	2+1+1	0.095, 0.081, 0.069	Chiral and continuum extrapolations performed simultaneously by adding an $O(a^2)$ term to the chiral fits.	Relative scale set through w_0 or $M_{c's'}$, the mass of a fictitious meson made of valence quarks of mass $r_0 m_{s'} = 0.22$ and $r_0 m_{c'} = 2.4$. Absolute scale through f_π^{iso} .

Table 109: Lattice spacings and description of actions used in $N_f = 2 + 1 + 1$ determinations of the D - and D_s -meson decay constants.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
RQCD/ALPHA 24	[13]	2+1	0.098, 0.085, 0.075, 0.064, 0.049, 0.039	Terms up to a^4 (possibly with mass-dependent coefficients) are included in the chiral-continuum extrapolation.	t_0 is used as intermediate scale with the physical scale set by a combination of f_π and f_K in the isosymmetric limit.
ALPHA 23	[5]	2+1	0.085, 0.075, 0.064, 0.049	$\mathcal{O}(a^2)$ terms, with mass-dependent coefficients, are included in the chiral-continuum extrapolation.	t_0 is used as intermediate scale with the physical scale set by a combination of f_π and f_K in the isosymmetric limit.

Table 110: Lattice spacings and description of actions used in $N_f = 2 + 1$ determinations of the D - and D_s -meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
ETM 13F ETM 14E ETM 21B	[10–12]	2+1+1	–	The axial current is absolutely normalized.

Table 111: Operator renormalization in $N_f = 2+1+1$ determinations of the D - and D_s -meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
RQCD/ALPHA 24	[13]	2+1	SF	The axial current is nonperturbatively improved and renormalized.
ALPHA 23	[5]	2+1	–	The axial current is absolutely normalized.

Table 112: Operator renormalization in $N_f = 2 + 1$ determinations of the D - and D_s -meson decay constants.

Collab.	Ref.	N_f	Action	Description
ETM 13F ETM 14E ETM 21B	[10–12]	2+1+1	tmWil	$0.15 \lesssim am_h \lesssim 0.28$.

Table 113: Heavy-quark treatment in $N_f = 2 + 1 + 1$ determinations of the D - and D_s -meson decay constants.

Collab.	Ref.	N_f	Action	Description
RQCD/ALPHA 24	[13]	2+1	npSW	$0.1 \leq am_h \leq 0.3$.
ALPHA 23	[5]	2+1	tmWil on npSW	$0.13 \leq am_h \leq 0.26$

Table 114: Heavy-quark treatment in $N_f = 2 + 1$ determinations of the D - and D_s -meson decay constants.

C.4.1 Form factors for semileptonic decays of charmed hadrons

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 22 [14]		2+1+1	0.12, 0.088, 0.057, 0.042	Combined chiral-continuum extrapolation using SU(2) heavy-meson rooted staggered chiral perturbation theory.	Scale setting using gradient flow w_0 with physical scale from f_π .
Meinel 21B	[15]	2+1	0.0828(3), 0.1106(3)	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematics estimated by varying fit form.	Scale setting using Ω mass in Ref. [16].
HPQCD 21A	[17]	2+1+1	0.042, 0.06, 0.09, 0.12, 0.15	Modified z -expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence. Discretization effects assumed dominated by the charm scale. Discretization errors on form factors between 0.4% and 1.2% as a function of the momentum transfer.	Scale setting from f_π via the flow quantity w_0 [18–20].
Zhang 21	[21]	2+1	0.080, 0.11	Continuum extrapolation combined with fit to q^2 -dependence of form factors in a “modified” z -expansion. Systematics estimated from difference between extrapolated results and results at smallest lattice spacing, and difference between two current renormalization methods.	Set from Wilson-flow quantity w_0 .
HPQCD 20	[22]	2+1+1	0.06, 0.09, 0.12, 0.15	Modified z -expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence, and, for the heavy-HISQ spectator b quark, the dependence on $1/m_Q$. The analysis combines data with NRQCD b quarks and data with HISQ heavy quarks.	Scale setting from f_π via the flow quantity w_0 [18–20].

Table 115: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2 + 1 + 1$ determinations of form factors for semileptonic decays of charmed hadrons. For HPQCD 22, see Tab. 141.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
FNAL/MILC 22	[14]	2+1+1	135, 130, 134, 308	Combined chiral-continuum extrapolation using SU(2) heavy-meson rooted staggered chiral perturbation theory at NLO, including NNLO analytic terms.
Meinel 21B	[15]	2+1	303, 340	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematic uncertainty estimated by repeating fit with added higher-order terms.
HPQCD 21A	[17]	2+1+1	315, 329, 129, 132, 131	Modified z -expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence. Polynomial dependence on quark masses, supplemented by a pion chiral logarithm. Fit result compared with alternative approach based on cubic splines in q^2 .
Zhang 21	[21]	2+1	300, 290	Dependence on pion mass neglected. No estimate of resulting systematic uncertainty.
HPQCD 20	[22]	2+1+1	329, 316, 132/305, 131/305	Modified z -expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence, and, for the heavy-HISQ spectator b quark, the dependence on $1/m_Q$. The analysis combines data with NRQCD b quarks and data with HISQ heavy quarks.

Table 116: Chiral extrapolation/minimum pion mass in determinations of form factors for semileptonic decays of charmed hadrons. For actions with multiple species of pions, masses quoted are the RMS pion masses for $N_f = 2+1$ and the Goldstone mode mass for $N_f = 2+1+1$. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings. For HPQCD 22, see Tab. 142.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 22	[14]	2+1+1	5.76, 4.22/5.63, 2.74/3.65/5.47, 2.69	3.95, 3.72, 3.72, 4.20	Finite-volume effects removed by correction to chiral logs due to sums over discrete momenta; corrections are $\mathcal{O}(0.01)\%$ overall. Effect of frozen topological charge at finest lattice spacing also corrected using χ PT and found to be $\lesssim 0.03\%$.
Meinel 21B	[15]	2+1	2.7, 2.7	4.1, 4.6	Finite-volume effects not quantified. Effects from unstable $\Lambda^*(1520)$ not quantified.
HPQCD 21A	[17]	2+1+1	2.73, 2.72, 2.81/5.62, 2.93/5.87, 2.45/4.89	$\gtrsim 3.7$	Finite-volume correction included in chiral fit, claimed to be a negligible effect. Effect of frozen topology in finest ensemble not discussed.
Zhang 21	[21]	2+1+1	2.6, 2.6	$\gtrsim 3.8$	No discussion of finite-volume effects.
HPQCD 20	[22]	2+1+1	2.72, 2.81, 2.93/5.87, 2.45/4.89	$\gtrsim 3.8$	Physical point ensemble at $a \simeq 0.15$ fm has $m_\pi L = 3.3$; the statement $m_\pi L \gtrsim 3.8$ applies to the other five ensembles.

Table 117: Finite-volume effects in determinations of form factors for semileptonic decays of charmed hadrons. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted. For HPQCD 22, see Tab. 143.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 22	[14]	2+1+1	NPR	Nonperturbative renormalization by imposing the PCVC relation.
Meinel 21B	[15]	2+1	mNPR	Residual matching factors ρ computed at 1-loop for vector and axial-vector currents, but at tree-level only for tensor currents. A systematic uncertainty is assigned to $\rho_{T^{\mu\nu}}$ as the double of $\max(\rho_{A^\mu} - 1 , \rho_{V^\mu} - 1)$.
HPQCD 21A	[17]	2+1+1	NP	Vector current normalized by imposing Ward identity at zero recoil.
Zhang 21	[21]	2+1	NP	Local vector current renormalized using ratio to conserved vector current. Axial current renormalized using ratio of off-shell quark matrix elements.
HPQCD 20	[22]	2+1+1	NP	Vector current normalized by imposing Ward identity at zero recoil.

Table 118: Operator renormalization in determinations of form factors for semileptonic decays of charmed hadrons. For HPQCD 22, see Tab. 144.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 22	[14]	2+1+1	HISQ	Valence heavy-quark masses range from 0.9 to 2 times the physical charm mass, with $0.164 \leq am_h \leq 0.8935$
Meinel 21B	[15]	2+1	Columbia RHQ for both the b and c quarks.	Discretization errors discussed as part of combined chiral-continuum- w fit. Higher-order fit also includes $\mathcal{O}(\alpha_s a \mathbf{p})$ terms to account for missing radiative corrections to $\mathcal{O}(a)$ improvement of the currents.
HPQCD 21A	[17]	2+1+1	HISQ	Bare charm-quark mass $0.194 \lesssim am_c \lesssim 0.8605$.
Zhang 21	[21]	2+1+1	SW	Bare charm-quark mass $0.235 \lesssim am_c \lesssim 0.485$. No $\mathcal{O}(a)$ improvement of currents.
HPQCD 20	[22]	2+1+1	Charm: HISQ Bottom (spectator): HISQ and NRQCD	Bare charm-quark HQS mass $0.274 \lesssim am_c \lesssim 0.827$. Bare bottom-quark HQS mass $0.274 \lesssim am_b \lesssim 0.8$.

Table 119: Heavy-quark treatment in determinations of form factors for semileptonic decays of charmed hadrons. For HPQCD 22, see Tab. 145.

C.5 Notes to Sec. 8 on B -meson decay constants, mixing parameters and form factors

C.5.1 $B_{(s)}$ -meson decay constants

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
Frezzotti 24	[23]	2+1+1	175, 140, 137, 141	One light-quark mass per lattice spacing. Chiral effects expected to be subdominant compared to other effects.

Table 120: Chiral extrapolation/minimum pion mass in determinations of the B - and B_s -meson decay constants for $N_f = 2+1+1$ simulations. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
QCDSF/UKQCD /CSSM 22	[24]	2+1	280, 155, 226, 290	Between one and three light-quark masses per lattice spacing. Generic fits to $(M_\pi^2/X_\pi^2 - 1)^2$ and $a^2(M_\pi^2/X_\pi^2 - 1)$ in the combined chiral-continuum extrapolation, with systematic errors estimated to be from 1.3% in f_{B_s}/f_B .
RBC/UKQCD 22	[25]	2+1	340, 302, 267, 371	Between one and three light-quark masses per lattice spacing. Combined chiral-continuum extrapolation using NLO SU(2) Heavy-Meson χ PT. No explicit estimate of systematic errors.

Table 121: Chiral extrapolation/minimum pion mass in determinations of the B - and B_s -meson decay constants for $N_f = 2+1$ simulations. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
Frezzotti 24	[23]	2+1+1	4.36, 5.09, 5.46, 5.46	3.9, 3.6, 3.8, 3.9	Finite-volume effects estimated to be subdominant to other sources of uncertainty, based in part on calculations in a larger ensemble in [26]

Table 122: Finite-volume effects in determinations of the B - and B_s -meson decay constants for $N_f = 2+1+1$ simulations. Each L -entry corresponds to a different lattice spacing.

Collab.	Ref.	N_f	L [fm]	$M_{\pi, \min} L$	Description
QCDSF/UKQCD /CSSM 22	[24]	2+1	2.62, 2.36/3.55, 3.26/4.35, 2.83	3.86, 3.10/4.07, 4.37/3.42, 4.03	Final result for f_{B_s}/f_B includes fits to ensembles with $M_{\pi} L > 4$. No explicit estimate of FV effects.
RBC/UKQCD 22	[25]	2+1	2.65, 2.65, 3.40, 2.00	4.57, 4.06, 4.60, 3.77	No explicit estimate of FV effects.

Table 123: Finite-volume effects in determinations of the B - and B_s -meson decay constants for $N_f = 2 + 1$ simulations. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
Frezzotti 24	[23]	2+1+1	0.091, 0.080, 0.068, 0.057	Continuum extrapolation linear in a^2 .	Scale set by w_0 , with details described in Ref. [27].

Table 124: Continuum extrapolations/estimation of lattice artifacts in determinations of the B - and B_s -meson decay constants for $N_f = 2 + 1 + 1$ simulations.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
QCDSF/UKQCD /CSSM 22	[24]	2+1	0.082, 0.074, 0.068, 0.059	Combined continuum and chiral extrapolation omits the term linear in a^2 . Systematic errors associated with discretization effects subdominant in f_{B_s}/f_B .	Scale setting procedure and scale uncertainty are not discussed.
RBC/UKQCD 22	[25]	2+1	0.11, 0.083, 0.071, 0.063	Combined continuum and chiral extrapolation includes term linear in a^2 . No estimate of systematic errors associated with discretization effects.	Scale setting procedure and scale uncertainty are not discussed.

Table 125: Continuum extrapolations/estimation of lattice artifacts in determinations of the B and B_s meson decay constants for $N_f = 2 + 1$ simulations.

Collab.	Ref.	N_f	Ren.	Description
Frezzotti 24	[23]	2+1+1	–	Nonperturbative operator renormalization provided by ETMC by private communication and unpublished at the time of this review.

Table 126: Description of the renormalization/matching procedure adopted in the determinations of the B - and B_s -meson decay constants for $N_f = 2 + 1 + 1$ simulations.

Collab.	Ref.	N_f	Ren.	Description
QCDSF/UKQCD /CSSM 22	[24]	2+1	mNPR	Operator renormalization is calculated partially non-perturbatively as $Z_{B_q} = \rho_A^{bq} \sqrt{Z_V^{bb} Z_V^{qq}}$, with perturbative contribution neglected, $\rho_A^{bq} = 1$.
RBC/UKQCD 22	[25]	2+1	mNPR	Operator renormalization is calculated partially non-perturbatively as $Z_{B_q} = \rho_A^{bq} \sqrt{Z_V^{bb} Z_V^{qq}}$.

Table 127: Description of the renormalization/matching procedure adopted in the determinations of the B - and B_s -meson decay constants for $N_f = 2 + 1$ simulations.

Collab.	Ref.	N_f	Action	Description
Frezzotti 24	[23]	2+1+1	tmWil	Heavy-strange meson extrapolated to physical B_s mass using HQET scaling linear in B/m_{H_s} , with contributions from QCD-HQET current matching and HQET axial current anomalous dimension at 1-loop.

Table 128: Heavy-quark treatment in determinations of the B - and B_s -meson decay constants for $N_f = 2 + 1 + 1$ simulations.

Collab.	Ref.	N_f	Action	Description
QCDSF/UKQCD /CSSM 22	[24]	2+1	RHQ	HQ tuning effects are estimated to be 0.06% in f_{B_s}/f_B . HQ discretization effects not explicitly estimated, although the continuum-limit fits do not indicate a strong a^2 dependence.
RBC/UKQCD 22	[25]	2+1	RHQ	HQ tuning and discretization effects not explicitly estimated. HQ tuning of new finest ensemble ongoing.

Table 129: Heavy-quark treatment in $N_f = 2 + 1$ determinations of the B - and B_s -meson decay constants.

C.5.2 $B_{(s)}$ -meson mixing matrix elements

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 19A [28]		2+1+1	0.15, 0.12, 0.09	Discretization errors start from $\alpha_s a^2$ and are included in the systematic error. It is estimated as 1.8% for individual bag parameters. Residual $\alpha_s a^2$ and a^4 errors from wrong-spin contributions are subtracted by including them in the chiral fit.	Scale setting done using Υ and Υ' mass splitting [29].

Table 130: Continuum extrapolations/estimation of lattice artifacts in determinations of the neutral B -meson mixing matrix elements for $N_f = 2 + 1 + 1$ simulations.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
RBC/UKQCD 18A [30]		2+1	0.11, 0.08, 0.07	Combined continuum (a^2) and heavy quark ($1/m_H$) extrapolation with the LO pion mass dependence (m_π^2) in the global fit.	Lattice scale and target quark masses are set using Ω , K and π masses [16, 31, 32].

Table 131: Continuum extrapolations/estimation of lattice artifacts in determinations of the neutral B -meson mixing matrix elements for $N_f = 2 + 1$ simulations.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
HPQCD 19A	[28]	2+1+1	311, 241, –	Pion mass in the Goldstone channel is as small as 130 MeV for two coarser lattices. NLO HMrS χ PT is used with NNLO analytic terms and other discretization errors. Staggered wrong-spin contributions are included.
RBC/UKQCD 18A	[30]	2+1	139, 139, 234	Combined continuum (a^2) and heavy quark ($1/m_H$) extrapolation with the LO pion mass dependence (m_π^2) in the global fit.

Table 132: Chiral extrapolation/minimum pion mass in determinations of the neutral B -meson mixing matrix elements. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi, \min} L$	Description
HPQCD 19A	[28]	2+1+1	2.4/3.5/4.6, 2.9/3.8/5.7, 2.8	7.3, 7.0, -	FV error is estimated to be negligible from FV HM χ PT.
RBC/UKQCD 18A	[30]	2+1	2.7/5.5, 2.6/5.3, 3.5	3.9, 3.8, 4.0	FV error is estimated to be less than 0.18% for SU(3)-breaking ratios from FV HM χ PT.

Table 133: Finite-volume effects in determinations of the neutral B -meson mixing matrix elements. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available).

Collab.	Ref.	N_f	Ren.	Description
HPQCD 19A	[28]	2+1+1	PT1 ℓ	HISQ-NRQCD 4-quark operators are matched through $O(1/M)$ and renormalized to 1-loop: included are those of $O(\alpha_s)$, $O(\Lambda_{\text{QCD}}/M)$, $O(\alpha_s/aM)$, $O(\alpha_s \Lambda_{\text{QCD}}/M)$. Remnant error is dominated by $O(\alpha_s \Lambda_{\text{QCD}}/M)$ 2.9% and $O(\alpha_s^2)$ 2.1% for individual bag parameters. Associated error for their SU(3) breaking ratio are negligible.
RBC/UKQCD 18A	[30]	2+1	-	Operators are renormalized multiplicatively due to chiral symmetry of DWF. No need to calculate the renormalization factor since only the SU(3) breaking ratios are examined.

Table 134: Operator renormalization in determinations of the neutral B -meson mixing matrix elements.

Collab.	Ref.	N_f	Action	Description
HPQCD 19A	[28]	2+1+1	NRQCD	See the entry in Tab. 134.
RBC/UKQCD 18A	[30]	2+1	DWF	Domain-wall fermion with 3 stout-smearing extends the reach to heavy mass, allowing to simulate up to half of the b-quark mass. Heavy mass errors on ξ are estimated as 0.8% from fitting range and 0.4% from higher order ($1/M^2$) by power counting.

Table 135: Heavy-quark treatment in determinations of the neutral B -meson mixing matrix elements.

C.5.3 Form factors entering determinations of $|V_{ub}|$ ($B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, $\Lambda_b \rightarrow p \ell \bar{\nu}$)

Collab.	Ref.	N_f	a [fm]		Continuum extrapolation	Scale setting
RBC/UKQCD 23	[33]	2+1	0.071, 0.083, 0.11		Joint chiral-continuum extrapolation using SU(2) hard-pion HM χ PT. Systematic uncertainty estimated by varying fit ansatz and form of coefficients, as well as implementing different cuts on data.	Scale implicitly set in the light-quark sector using the Ω^- mass, cf. [16, 30, 32].
JLQCD 22	[34]	2+1	0.044, 0.055, 0.080		Discretization effects treated using overall factors of $(1 + C_{a^2}(\Lambda_{\text{QCD}}a)^2 + C_{(am_Q)^2}(am_Q)^2)$, with independent coefficients for the two form factors. Systematic uncertainties estimated by adding $C_{a^4}(\Lambda_{\text{QCD}}a)^4$ or $C_{(am_Q)^4}(am_Q)^4$ terms.	Relative scale set using gradient-flow time $t_0^{1/2}/a$. Absolute scale $t_0^{1/2}$ taken from Ref. [35].
FNAL/MILC 19	[36]	2+1	0.06, 0.12	0.09,	HM χ PT expansion used at next-to-leading order in SU(2) and leading order in $1/M_B$, including next-to-next-to-leading-order (NNLO) analytic and generic discretization terms. Hard kaons assumed to decouple. Systematic uncertainties estimated by varying fit ansatz and data range. The (stat + chiral extrap + HQ discretization + g_π) uncertainty dominates the error budget, ranging from 2–3% at $q^2 \gtrsim 21 \text{ GeV}^2$ to up to 8–10% in the lower end of the accessed q^2 interval.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 , including related uncertainty estimates, taken from [37].

Table 136: Continuum extrapolations/estimation of lattice artifacts in determinations of $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, and $\Lambda_b \rightarrow p \ell \bar{\nu}$ form factors.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RBC/UKQCD 23	[33]	2+1	268, 301, 340	Joint chiral-continuum extrapolation using SU(2) hard-pion HM χ PT. Systematic uncertainty estimated by varying fit ansatz and form of coefficients, as well as implementing different cuts on data.
JLQCD 22	[34]	2+1	300, 300, 230	Chiral extrapolation uses SU(2) hard-pion heavy-meson chiral perturbation theory at next-to-leading order. Systematic uncertainty estimated by adding M_π^4 terms or by making the coefficients of the chiral logs fit parameters.
FNAL/MILC 19	[36]	2+1	255, 277, 456	HMrS χ PT expansion used at next-to-leading order in SU(2) and leading order in $1/M_B$, including next-to-next-to-leading-order (NNLO) analytic and generic discretization terms. Hard kaons assumed to decouple. Systematic uncertainties estimated by varying fit ansatz and data range.

Table 137: Chiral extrapolation/minimum pion mass in determinations of $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, and $\Lambda_b \rightarrow p \ell \bar{\nu}$ form factors. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi, \min} L$	Description
RBC/UKQCD 23	[33]	2+1	3.4, 2.7, 2.6	4.6, 4.0, 4.4	Finite-volume effects removed by correction to chiral logs due to sums over discrete momenta; quoted maximum corrections are 0.13% for f_+ and 0.06% for f_0 .
JLQCD 22	[34]	2+1	2.6, 3.9	$\gtrsim 4.0$	Finite-volume effects in form factors deemed negligible. Bias in pion mass due to topology freezing at finest lattice spacing estimated to be $\sim 0.1\%$.
FNAL/MILC 19	[36]	2+1	3.8, 2.5/2.9/3.6/5.8, 2.9	$\gtrsim 3.8$	Finite-volume effects estimated by comparing infinite volume integrals with finite sums in HMrS χ PT, found to be negligible.

Table 138: Finite-volume effects in determinations of $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, and $\Lambda_b \rightarrow p \ell \bar{\nu}$ form factors. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
RBC/UKQCD 23	[33]	2+1	mNPR	Perturbative truncation error estimated as full size of $O(\alpha_s)$ correction at the 0.083 fm lattice spacing.
JLQCD 22	[34]	2+1	NPR	$Z_{V_{qq}}$ obtained using position-space current-current correlators. For heavier quark masses, $\sqrt{Z_{V_{QQ}} Z_{V_{qq}}}$ is used, where $Z_{V_{QQ}}$ is the renormalization factor of the flavour-conserving temporal vector current, determined using charge conservation.
FNAL/MILC 19	[36]	2+1	mNPR	Perturbative truncation error estimated at 1% with size of 1-loop correction on next-to-finest ensemble.

Table 139: Operator renormalization in determinations of $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, and $\Lambda_b \rightarrow p \ell \bar{\nu}$ form factors.

Collab.	Ref.	N_f	Action	Description
RBC/UKQCD 23	[33]	2+1	Columbia RHQ	Heavy-quark discretization errors estimated by power counting.
JLQCD 22	[34]	2+1	DWF	Bare heavy-quark masses satisfy $am_Q < 0.7$ and reach from the charm mass up to 2.44 times the charm mass. Form factors extrapolated linearly in $1/m_Q$ to the bottom mass.
FNAL/MILC 19	[36]	2+1	Fermilab	(See comments for continuum limit extrapolation.)

Table 140: Heavy-quark treatment in determinations of $B \rightarrow \pi \ell \nu$, $B_s \rightarrow K \ell \nu$, and $\Lambda_b \rightarrow p \ell \bar{\nu}$ form factors.

C.5.4 Form factors for rare decays of beauty hadrons

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 22	[38]	2+1+1	0.15, 0.12, 0.090, 0.088, 0.059, 0.044	Combined extrapolation in lattice spacing, light-quark mass, strange-quark mass, heavy-quark mass, and momentum transfer using modified z expansion. Stability tested by varying fit form, changing prior widths, and removing data subsets.	Scale setting using gradient flow w_0 with physical scale from f_π .
Meinel 20, Meinel 21B	[15, 39]	2+1	0.0828(3), 0.1106(3)	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematic uncertainty estimated by repeating fit with added higher-order terms.	Scale setting using Ω mass in Ref. [16].

Table 141: Continuum extrapolations/estimation of lattice artifacts in determinations of form factors for rare decays of beauty hadrons.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
HPQCD 22	[38]	2+1+1	131, 132, 313, 128, 325, 308	Combined extrapolation in lattice spacing, light-quark mass, strange-quark mass, heavy-quark mass, and momentum transfer using modified z expansion. Logarithms from hard-pion χ PT included. Stability tested by varying fit form, changing prior widths, and removing data subsets.
Meinel 20, Meinel 21B	[15, 39]	2+1	303, 340	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematic uncertainty estimated by repeating fit with added higher-order terms.

Table 142: Chiral extrapolation/minimum pion mass in determinations of form factors for rare decays of beauty hadrons. For actions with multiple species of pions, masses quoted are the RMS pion masses for $N_f = 2 + 1$ and the Goldstone mode mass for $N_f = 2 + 1 + 1$. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi, \min} L$	Description
HPQCD 22	[38]	2+1+1	2.4/4.8, 2.88/5.76, 2.88, 5.63, 2.83, 2.82	3.19, 3.86, 4.57, 3.66, 4.67, 4.41	Finite-volume effects included in fit by replacing infinite-volume chiral logs with sums over discrete momenta.
Meinel 20, [15, 39] Meinel 21B		2+1	2.7, 2.7	4.1, 4.6	Finite-volume effects not quantified. Effects from unstable $\Lambda^*(1520)$ not quantified.

Table 143: Finite-volume effects in determinations of form factors for rare decays of beauty hadrons. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, masses quoted are the RMS pion masses for $N_f = 2 + 1$ and the Goldstone mode mass for $N_f = 2 + 1 + 1$.

Collab.	Ref.	N_f	Ren.	Description
HPQCD 22	[38]	2+1+1	NPR	Z_V and Z_A obtained from Ward identities. Z_T determined using RI-SMOM.
Meinel 20, [15, 39] Meinel 21B		2+1	mNPR	Residual matching factors ρ computed at 1-loop for vector and axial-vector currents, but at tree-level only for tensor currents. A systematic uncertainty is assigned to $\rho_{T^{\mu\nu}}$ as the double of $\max(\rho_{A^\mu} - 1 , \rho_{V^\mu} - 1)$.

Table 144: Operator renormalization in determinations of form factors for rare decays of beauty hadrons.

Collab.	Ref.	N_f	Action	Description
HPQCD 22	[38]	2+1+1	HISQ	Extrapolation to the physical b -quark mass using terms with powers and logarithms of the inverse heavy-meson mass in the modified z -expansion fit. Heavy-quark masses in lattice units satisfy $am_h \leq 0.9$. The heavy-light pseudoscalar meson mass reaches $\approx 0.94 M_{B,\text{phys.}}$.
Meinel 20, Meinel 21B	[15, 39]	2+1	Columbia RHQ	Discretization errors discussed as part of combined chiral-continuum- w fit. Higher-order fit also includes $\mathcal{O}(\alpha_s a \mathbf{p})$ terms to account for missing radiative corrections to $\mathcal{O}(a)$ improvement of the currents.

Table 145: Heavy-quark treatment in determinations of form factors for rare decays of beauty hadrons.

C.5.5 Form factors entering determinations of $|V_{cb}|$ ($B_{(s)} \rightarrow D_{(s)}^{(*)}\ell\nu$, $\Lambda_b \rightarrow \Lambda_c^{(*)}\ell\bar{\nu}$) and $R(D_{(s)})$

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 23	[40]	2+1+1	0.044, 0.058, 0.088	Combined chiral-continuum and heavy-quark extrapolations using HMrS χ PT. The recoil dependence in powers of $(w - 1)$ is fitted using BGL-inspired coefficients. Zero-recoil uncertainty negligible compared to other sources of error.	Scale setting from Wilson flow, fixing the slope $t \frac{d}{dt} \{t^2 \langle E(t) \rangle\} _{t=w_0^2} = 0.3$, with w_0 taken from [18]. Uncertainty related to scale setting estimated at $\approx 0.5\%$.
JLQCD 23	[41]	2+1	0.044, 0.055, 0.080	Combined chiral-continuum and heavy-quark extrapolations using HM χ PT. Each form factor is extrapolated separately. Zero-recoil uncertainty estimated at $\approx 0.9\%$.	Scale setting from Wilson flow, fixing the slope $t \frac{d}{dt} \{t^2 \langle E(t) \rangle\} _{t=w_0^2} = 0.3$, with w_0 taken from [35]. Uncertainty related to scale setting estimated at $\approx 1.7\%$.
FNAL/MILC 21	[42]	2+1	0.045, 0.06, 0.09, 0.12, 0.15	Combined chiral-continuum extrapolation using HMrS χ PT. Total uncertainty quoted at 0.7%.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 , including related uncertainty estimates, taken from [37]. Uncertainty related to scale setting estimated at less than 0.1%.
Meinel 21, Meinel 21B	[15, 43]	2+1	0.0828(3), 0.1106(3)	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematics estimated by varying fit form.	Scale setting using Ω mass in Ref. [16].

Table 146: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2 + 1$ determinations of $B_{(s)} \rightarrow D_{(s)}^{(*)}\ell\nu$ and $\Lambda_b \rightarrow \Lambda_c^{(*)}\ell\bar{\nu}$ form factors, and of $R(D_{(s)})$.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
HPQCD 23	[40]	2+1+1	315, 135, 129	Combined chiral-continuum and heavy-quark extrapolations using HMrS χ PT. The recoil dependence in powers of $(w - 1)$ is fitted using BGL-inspired coefficients. Zero-recoil uncertainty negligible compared to other sources of error.
JLQCD 23	[41]	2+1	284, 300, 226	Combined chiral-continuum and heavy-quark extrapolations using HMrS χ PT. Each form factor is extrapolated separately. Zero-recoil uncertainty estimated at $\approx 0.9\%$.
FNAL/MILC 21	[42]	2+1	320, 220, 180, 270, 340	Combined chiral-continuum extrapolation using HMrS χ PT. Systematic errors estimated by adding higher-order analytic terms and varying the D^*-D - π coupling. Total uncertainty quoted at 0.7%.
Meinel 21, Meinel 21B	[15, 43]	2+1	303, 340	Combined chiral-continuum extrapolation as part of the expansion of form factor shape in powers of $w - 1$. Systematic uncertainty estimated by repeating fit with added higher-order terms.

Table 147: Chiral extrapolation/minimum pion mass in $N_f = 2 + 1$ determinations of $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$ and $\Lambda_b \rightarrow \Lambda_c^{(*)} \ell \bar{\nu}$ form factors, and of $R(D_{(s)})$. For actions with multiple species of pions, masses quoted are the RMS pion masses for $N_f = 2 + 1$ and the Goldstone mode mass for $N_f = 2 + 1 + 1$. The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
HPQCD 23	[40]	2+1+1	2.8, 2.8–5.5, 2.8–5.6	4.5, 3.7, 3.7	Finite-volume effects expected to be negligible, including the effect of frozen topology in the finest ensemble, according to [14].
JLQCD 23	[41]	2+1	2.8, 2.6, 2.6– 3.9	4.0, 4.0, 4.4	Study effects of topology freezing and compare ensembles with similar parameters but different volumes.
FNAL/MILC 21	[42]	2+1	4.6, 4.3–6.3, 4.1–5.8, 3.8– 6.2, 3.9	$\gtrsim 3.8$	Finite-volume error estimated to be negligible at zero recoil using HMrS χ PT. Given the values $m_\pi L \gtrsim 3.7$ and the smallness of the chiral logs, expectations are that finite-volume errors remain negligible in the whole recoil range.
Meinel 21, Meinel 21B	[15, 43]	2+1	2.7, 2.7	4.1, 4.6	Finite-volume effects not quantified. Effects from unstable Λ_c^* not quantified.

Table 148: Finite-volume effects in determinations of $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$ and $\Lambda_b \rightarrow \Lambda_c^{(*)} \ell \bar{\nu}$ form factors, and of $R(D_{(s)})$. Each L -entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
HPQCD 23	[40]	2+1+1	NPR	Vector (axial) currents renormalized non-perturbatively using the PCVC (PCAC) relation.
JLQCD 23	[41]	2+1	mNPR	Majority of current renormalization factor cancels in ratio of lattice correlation functions. Remaining correction expected to behave better than $O(a)$, and vanishes in the continuum limit.
FNAL/MILC 21	[42]	2+1	mNPR	Majority of current renormalization factor cancels in double ratio of lattice correlation functions. Remaining correction calculated with 1-loop tadpole-improved lattice perturbation theory. Systematic uncertainty estimated at 0.1%.
Meinel 21, Meinel 21B	[15, 43]	2+1	mNPR	Residual matching factors ρ computed at 1-loop for vector and axial-vector currents, but at tree-level only for tensor currents. A systematic uncertainty is assigned to $\rho_{T\mu\nu}$ as the double of $\max(\rho_{A\mu} - 1 , \rho_{V\mu} - 1)$.

Table 149: Operator renormalization in determinations of $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$ and $\Lambda_b \rightarrow \Lambda_c^{(*)} \ell \bar{\nu}$ form factors, and of $R(D_{(s)})$.

Collab.	Ref.	N_f	Action	Description
HPQCD 23	[40]	2+1+1	HISQ for both the b and c quarks.	Values of bare heavy-quark masses up to $am_h = 0.8$. The error from continuum limit and extrapolation to physical b mass at zero recoil is quite small, but it becomes dominant at mid-recoil.
JLQCD 23	[41]	2+1	Möbius Domain-Wall for both the b and c quarks.	Values of bare heavy-quark masses up to $am_h = 0.69$. The systematics associated to the extrapolation to physical b mass stays under 4% for all form factors in the whole recoil range.
FNAL/MILC 21	[42]	2+1	Fermilab RHQ for both the b and c quarks.	Discretization errors discussed as part of combined chiral-continuum stemming from $\alpha_s a$, a^2 and a^3 terms.
Meinel 21, Meinel 21B	[15, 43]	2+1	Columbia RHQ for both the b and c quarks.	Discretization errors discussed as part of combined chiral-continuum- w fit. Higher-order fit also includes $\mathcal{O}(\alpha_s a \mathbf{p})$ terms to account for missing radiative corrections to $\mathcal{O}(a)$ improvement of the currents.

Table 150: Heavy-quark treatment in determinations of $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \nu$ and $\Lambda_b \rightarrow \Lambda_c^{(*)} \ell \bar{\nu}$ form factors, and of $R(D_{(s)})$.

C.6 Notes to Sec. 9 on the strong coupling α_s

C.6.1 Renormalization scale and perturbative behaviour

Collab.	Ref.	N_f	α_{eff}	n_l	Description
Hasenfratz 23	[44]	0	$0.095 < \alpha_{\text{eff}} < 1.26$	2	GF scheme does not reach perturbative asymptotics.
Wong 23	[45]	0	$0.095 < \alpha_{\text{eff}} < 1.26$	2	GF scheme does not reach perturbative asymptotics.
Brambilla 23	[46]	0	$0.27 < \alpha_{\text{eff}} < 0.36$	3	Static force using operator insertion.
Chimirri 23	[47]	0	$0.17 < \alpha_{\overline{\text{MS}}} < 0.25$	2	Values for α read off from figure.
Bribian 21	[48]	0	SF: 0.07–0.19 TGF: 0.05–0.92	2	Step scaling with TGF, nonpert. matching to SF.

Table 151: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 0$.

Collab.	Ref.	N_f	α_{eff}	n_l	Description
ALPHA 22	[49]	2+1, 0	0.08–0.95	2	Decoupling $N_f = 3 \rightarrow N_f = 0$; uses $N_f = 0$ step-scaling from Dalla Brida 19 [50].

Table 152: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 3$.

C.6.2 Continuum limit

Collab.	Ref.	N_f	$a\mu$	Description
Hasenfratz 23	[44]	0	$0.158 < a/\sqrt{8t} < 0.29$	GF scheme, infinite-volume extrapolation, direct determination of the β -function.
Wong 23	[45]	0	$0.16 < a/\sqrt{8t} < 0.28$	GF scheme, infinite-volume extrapolation, direct determination of the β -function.
Brambilla 23	[46]	0	$0.23 < a\mu < 0.49$	Force between static quarks using operator insertion.
Chimirri 23	[47]	0	$0.8m_c < \mu < 3.5m_c$	Lattice spacings a in the range 0.01–0.07 fm; Scale defined by $\mu = s \times \overline{m}_{\overline{\text{MS}},c}(\mu)$.
Bribian 21	[48]	0	TGF: $0.041 < a\mu < 0.083$ SF: $0.063 < a\mu < 0.17$	Step scaling TGF scheme, nonpert. matching to SF scheme.

Table 153: Continuum limit for α_s determinations with $N_f = 0$.

Collab.	Ref.	N_f	$a\mu$	Description
ALPHA 22	[49]	2+1, 0	$0.021 < a\mu_{\text{dec}} < 0.083$	Decoupling $N_f = 3 \rightarrow N_f = 0$; continuum limit subject to cutoff $aM < 0.4$, $z = M/\mu_{\text{dec}} = 4 - 12$.

Table 154: Continuum limit for α_s determinations with $N_f = 3$.

C.7 Notes to Sec. 10 on nucleon matrix elements

Collab.	Ref.	N_f	a [fm]	Description
ETM 23	[51]	2+1+1	0.057, 0.069, 0.080	Extrapolation via a fit which is linear in a^2 .
PNDME 23	[52]	2+1+1	0.06, 0.09, 0.12, 0.15	Physical-point extrapolations performed simultaneously, keeping only the leading-order terms in the various expansion parameters.
ETM 22	[53]	2+1+1	0.057, 0.069, 0.080	Extrapolation via a fit which is linear in a^2 .

Table 155: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Mainz 24	[54]	2+1	0.049,0.064,0.076,0.086	Extrapolation performed as part of a simultaneous fit in a , M_π and $M_\pi L$.
PACS 23	[55]	2+1	0.063, 0.085	Discretization effects estimated by difference between two ensembles.
RQCD 23	[56]	2+1	0.039, 0.049, 0.064, 0.076, 0.086, 0.098	Extrapolation performed using terms up to a^2 in the lattice spacing.
QCDSF/ UKQCD/ CSSM 23	[57]	2+1	0.052, 0.059, 0.068, 0.074, 0.082	Extrapolation performed including leading discretization effects.
PACS 22B	[58]	2+1	0.085	Single lattice spacing.
Mainz 22	[59]	2+1	0.049,0.064,0.076,0.086	Extrapolation performed as part of a simultaneous fit in a , M_π and $M_\pi L$.

Table 156: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 23	[51]	2+1+1	140, 138, 141	Three pion masses within 3% of the physical value.
PNDME 23	[52]	2+1+1	321, 228, 138, 136	Fit performed including leading-order pion-mass dependence.
ETM 22	[53]	2+1+1	140, 138, 141	Three pion masses within 3% of the physical value.

Table 157: Chiral extrapolation/minimum pion mass in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
Mainz 24	[54]	2+1	176, 130,218,228	Physical-point extrapolations were performed simultaneously in the lattice spacing, pion mass, and volume.
PACS 23	[55]	2+1	138, 135	Three near-physical pion masses.
RQCD 23	[56]	2+1	336,176, 131,156,127,338	Extrapolations performed using leading-order chiral expressions for the pion mass.
QCDSF/ UKQCD/ CSSM 23	[57]	2+1	290,315,270,220,280,347	Combined pion-mass, lattice-spacing, and volume extrapolations, performed around chiral SU(3) point.
PACS 22B	[58]	2+1	135	Two near-physical pion masses.
Mainz 22	[59]	2+1	176, 130,218,228	Physical-point extrapolations were performed simultaneously in the lattice spacing, pion mass, and volume.

Table 158: Chiral extrapolation/minimum pion mass in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 23	[51]	2+1+1	5.1, 5.4, 5.5	3.6, 3.8, 3.9	No extrapolation performed.
PNDME 23	[52]	2+1+1	2.4, 2.9-4.8, 2.9-5.8, 2.9-5.8	3.62, 2.98	Physical-point extrapolations performed simultaneously, using the leading-order terms in the various expansion parameters.
ETM 22	[53]	2+1+1	5.1, 5.4, 5.5	3.62, 2.98	No extrapolation performed.

Table 159: Finite-volume effects in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
Mainz 24	[54]	2+1	4.1, 4.8, 6.1, 4.7	4.7, 5.4, 4.0, 4.2	Extrapolation performed including a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a , M_π and $M_\pi L$.
PACS 23	[55]	2+1	5.5-10.9, 10.9	3.8-7.5, 7.5	Negligible finite-volume effects seen between volumes.
RQCD 23	[56]	2+1	4.8, 2.75-11, 4.9-9.7, 4.1-12.3, 6.3-9.4, 7.5	4.0, 3.5, 3.8, 4.1, 4.3, 4.3	Leading asymptotic form for finite-volume corrections used for extrapolation.
QCDSF/ UKQCD/ CSSM 23	[57]	2+1	2.6, 2.4, 3.3-4.4, 2.8, 2.5	4.0, 3.9	Combined pion-mass, lattice-spacing, and volume extrapolations performed.
PACS 22B	[58]	2+1	5.5-10.9	3.8-7.5	Negligible finite-volume effects seen between volumes.
Mainz 22	[59]	2+1	4.1, 4.8, 6.1, 4.7	4.7, 5.4, 4.0, 4.2	Extrapolation performed including a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a , M_π and $M_\pi L$.

Table 160: Finite-volume effects in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	Ren.
ETM 23	[51]	2+1+1	RI'-MOM
PNDME 23	[52]	2+1+1	RI-SMOM
ETM 22	[53]	2+1+1	RI'-MOM

Table 161: Renormalization in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	Ren.
Mainz 24	[54]	2+1	RI-SMOM
PACS 23	[55]	2+1	RI-SMOM
RQCD 23	[56]	2+1	RI'-SMOM
QCDSF/ UKQCD/ CSSM 23	[57]	2+1	RI'-MOM
PACS 22B	[58]	2+1	RI-SMOM
Mainz 22	[59]	2+1	RI-SMOM

Table 162: Renormalization in determinations of the isovector axial, scalar and tensor charges with $2 + 1$ quark flavours.

Collab.	Ref.	N_f	τ [fm]	Description
ETM 23	[51]	2+1+1	[0.6–1.6] [0.6–1.5] [0.5–1.2]	Compared results from the plateau, summation method and two-state fits.
PNDME 23	[52]	2+1+1	[0.8–1.4] [1–1.4, 1–1.4, 1–1.4, 1–1.7] [0.9–1.4, 0.9–1.4, 0.7–1.4] [1–1.4, 1.1–1.4, 1–1.4, 1.1–1.4, 1–1.3]	Several strategies to remove excited-state contributions, including removing $N\pi$ contributions.
ETM 22	[53]	2+1+1	[0.6–1.6] [0.6–1.5]	Compared results from the plateau, summation method and two-state fits.

Table 163: Control of excited-state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
PACS 22B	[58]	2+1	[0.9–1.4]	Excited-state contributions estimated using different time separations and smearings.
Mainz 22	[59]	2+1	[0.4–1.5, 0.4–1.5, 0.4–1.5, 0.4–1.5] [1–1.2] [0.3–1.5, 0.3–1.5, 0.3–1.5] [0.3–1.4, 0.3–1.4, 0.3–1.4, 0.3–1.4, 0.3–1.4] [0.2–1.4, 0.2–1.4, 0.2–1.4] [0.3–1.4] [0.9–1.5]	Two-state fits to the summation method.

Table 164: (cntd.) Control of excited-state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	a [fm]	Description
PNDME 21	[60]	2+1+1	0.12,0.09,0.06	Joint continuum and chiral fit includes a aM_π^2 term.
Mainz 23	[61]	2+1	0.08,0.07,0.06,0.05	Joint continuum, chiral and finite-volume (correlated) fit of $\sigma_{\pi N}$ and σ_s includes a $aM_{\pi,K}^2$ term. Fits are performed including and excluding this term.

Table 165: Continuum extrapolation/estimation of lattice artifacts in direct determinations of $\sigma_{\pi N}$ and σ_s .

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
PNDME 21	[60]	2+1+1	228,138,235	Joint continuum and chiral fit including the SU(2) NNLO baryon χ PT [62] terms.
Mainz 23	[61]	2+1	219, 154, 128, 174	Joint continuum, chiral and finite-volume (correlated) fit of $\sigma_{\pi N}$ and σ_s utilizing SU(3) NNLO covariant baryon χ PT with the EOMS loop regularization scheme [63–65]. Cuts on the pion mass excluding ensembles with $M_\pi > 220, 285$ or 360 MeV are made.

Table 166: Chiral extrapolation/minimum pion mass in direct determinations of $\sigma_{\pi N}$ and σ_s .

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PNDME 21	[60]	2+1+1	2.9–3.8 2.8–5.6 2.8–3.7	4.4, 3.9 4.4	Finite-volume terms are not included in the joint continuum-chiral extrapolation.
Mainz 23	[61]	2+1	2.7–4.1 2.4–4.8 2.0–6.1 2.4–4.7	4.6 3.8 4.0 4.2	Joint continuum, chiral and finite-volume (correlated) fit of $\sigma_{\pi N}$ and σ_s includes a term derived from the SU(2) finite-volume expression for the nucleon mass in Ref. [66]. Fits are performed including and excluding this term.

Table 167: Finite-volume effects in direct determinations of $\sigma_{\pi N}$ and σ_s .

Collab.	Ref.	N_f	Ren.	Description
PNDME 21	[60]	2+1+1	-/-	Flavour mixing occurs due to breaking of chiral symmetry. The ratio of Z^{ns}/Z^s is estimated to be close to 1 and the mixing is neglected.
Mainz 23	[61]	2+1	NP/NP	Flavour mixing occurs due to breaking of chiral symmetry. The mixing is implemented using ratios of the light- and strange-quark masses rather than utilising Z^{ns}/Z^s .

Table 168: Renormalization for direct determinations of $\sigma_{\pi N}$ and σ_s . The type of renormalization (Ren.) is given for $\sigma_{\pi N}$ first and σ_s second. The label 'na' indicates that no renormalization is required.

Collab.	Ref.	N_f	τ [fm]	Description
PNDME 21	[60]	2+1+1	[1.0–1.7, 1.0–1.7]/all [0.9–1.4, 0.9–1.4]/all [1.1–1.4, 1.1–1.4]/all	The two- and three-point functions are fitted simultaneously including four and three states, respectively. The final results are obtained using a narrow-width prior to set the first excited-state energy to that of the lowest $N\pi$ state.
Mainz 23	[61]	2+1	[0.3–1.5]/all [0.3–1.5]/all [0.3–1.4]/all [0.2–1.4]/all	Summation method including the ground-state terms and ratio fits including an excited state with the energy fixed with a prior to the lowest $N\pi$ energy are considered. The final result combines results from both fit types.

Table 169: Control of excited-state contamination in direct determinations of $\sigma_{\pi N}$ and σ_s . The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling. The range of τ for the connected (disconnected) contributions to the three-point correlation functions is given first (second). If a wide range of τ values is available this is indicated by “all” in the table.

Collab.	Ref.	N_f	a [fm]	Description
RQCD 22	[67]	2+1	0.10,0.09,0.08, 0.06,0.05,0.04	Combined continuum, chiral and volume fit to the baryon octet. Leading $O(a^2)$ terms are included in the parameterisation. Fits are performed also excluding the coarsest lattice spacing.

Table 170: Continuum extrapolations/estimation of lattice artifacts in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RQCD 22	[67]	2+1	338,127,216 131,176,336	Combined continuum, chiral and volume fit to the baryon octet. Fits utilizing SU(3) NNLO covariant baryon χ PT with the EOMS loop regularization scheme [63–65] are performed. Cuts on the flavour average meson mass squared are made.

Table 171: Chiral extrapolation/minimum pion mass in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min} L$	Description
RQCD 22	[67]	2+1	2.3, 2.0–5.5, 2.4–4.8, 2.0–6.1, 2.4–4.7 2.5	4.0, 3.5, 5.3, 4.0, 4.2, 4.2	Combined continuum, chiral and volume fit to the baryon octet. Finite-volume terms from NNLO covariant baryon χ PT (with no new fit parameters) are included in the fit [68–70]. Ensembles with $L < 2.3$ fm are excluded from the analysis.

Table 172: Finite-volume effects in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	a [fm]	Description
ETM 22	[53]	2+1+1	0.08,0.07,0.06	Linear extrapolation in a^2 .
PNDME 20A	[71]	2+1+1	0.15,0.12,0.09,0.06	Extrapolation performed using a linear term in a as part of a simultaneous fit in a , M_π and $M_\pi L$.
ETM 20C	[72]	2+1+1	0.08	Single lattice spacing.
ETM 19A	[73]	2+1+1	0.08	Single lattice spacing.

Table 173: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Mainz 24	[54]	2+1	0.09,0.08,0.06,0.05	A number of simultaneous a , M_π and $M_\pi L$ fits are performed using a linear term in a . The final results are obtained from a weighted average.
LHPC 24	[74]	2+1	0.12,0.09	A Bayesian fit is performed including an a and an a^2 term.
NME 21A	[75]	2+1	0.127, 0.09, 0.07	Extrapolation performed using a linear term in a as part of a simultaneous fit in a , M_π and $M_\pi L$.
NME 20	[76]	2+1	0.127, 0.09, 0.07	Extrapolation performed using a linear term in a as part of a simultaneous fit in a , M_π and $M_\pi L$.
Mainz 19	[77]	2+1	0.05,0.06,0.08,0.09	Extrapolation performed as part of a simultaneous fit in a , M_π and $M_\pi L$.
χ QCD 18A	[78]	2+1	0.143, 0.11, 0.114, 0.083	Partially quenched calculation. Extrapolation performed as part of a simultaneous fit in a^2 , M_π and $M_\pi L$ using expression from HBChPT.
LHPC 12A	[79]	2+1	0.12,0.09	No statistically significant discretization effects observed. Results assumed to be constant in a .
LHPC 10	[80]	2+1	0.12	Single lattice spacing.
RBC/UKQCD 10D [81]	[81]	2+1	0.11	Single lattice spacing.

Table 174: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
RQCD 18	[82]	2	0.081, 0.071, 0.060	No significant $O(a)$ effects observed.
ETM 17C	[83]	2	0.0938	Single ensemble.
ETM 15D	[84]	2	0.093	Single ensemble.
RQCD 14A	[85]	2	0.081, 0.071, 0.060	Analysis not conclusive.

Table 175: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 22	[53]	2+1+1	140,137,141	Simulate close to M_{π}^{phys} .
PNDME 20A	[71]	2+1+1	321,228,138,136	Extrapolation performed using a linear term in M_{π}^2 as part of a simultaneous fit in a , M_{π} and $M_{\pi}L$.
ETM 20C	[72]	2+1+1	139	Single pion mass within 3% of the physical value.
ETM 19A	[73]	2+1+1	139	Single pion mass within 3% of the physical value.

Table 176: Chiral extrapolation/minimum pion mass in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
Mainz 24	[54]	2+1	228,218,130,176	A number of simultaneous a , M_π and $M_\pi L$ fits are performed using terms which appear in the SU(2) NNLO ChPT expression for the axial charge and (alternatively) a linear term in M_π^2 . The final results are obtained from a weighted average.
LHPC 24	[74]	2+1	136,133	Simulate close to M_π^{phys} .
NME 21A	[75]	2+1	285, 169, 167	Extrapolation performed using a linear term in M_π^2 as part of a simultaneous fit in a , M_π and $M_\pi L$.
NME 20	[76]	2+1	285, 169, 167	Extrapolation performed using a linear term in M_π^2 as part of a simultaneous fit in a , M_π and $M_\pi L$.
Mainz 19	[77]	2+1	290, 200, 260	Extrapolation performed using logarithmic and quadratic terms in M_π as part of a simultaneous fit in a , M_π and $M_\pi L$.
χ QCD 18A	[78]	2+1	171,330,139,300	Partially quenched calculation. Extrapolation performed as part of a simultaneous fit in a^2 , M_π and $M_\pi L$ using expression from HBChPT.
LHPC 12A	[79]	2+1	149,317	Chiral fit formula based on the “small-scale expansion” to order ϵ^3 with some coefficients fixed.
LHPC 10	[80]	2+1	293	Chiral fit formula based on the “small-scale expansion” to order ϵ^3 with some coefficients fixed.
RBC/UKQCD 10D	[81]	2+1	329,416,555,668	Constant fit to heaviest three and linear fit to lightest two pion masses gives the quoted range.

Table 177: Chiral extrapolation/minimum pion mass in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
RQCD 18	[82]	2	280, 150, 260	Chiral fit using BChPT. Kept terms up to $O(M_\pi^3)$.
ETM 17C	[83]	2	≈ 135	Single ensemble within 3% of physical pion mass.
ETM 15D	[84]	2	131	Single ensemble within 3% of physical pion mass.
RQCD 14A	[85]	2	280, 150, 260	No significant dependence on pion mass observed.

Table 178: Chiral extrapolation/minimum pion mass in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 22	[53]	2+1+1	5.1, 5.5, 5.5	3.6, 3.8, 3.9	Finite-volume effects not estimated.
PNDME 20A	[71]	2+1+1	2.4, 4.8, 5.6, 5.5	3.9, 5.5, 3.9, 3.7	Fit performed using a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a , M_π and $M_\pi L$.
ETM 20C	[72]	2+1+1	5.1	3.6	Finite-volume effects not estimated.
ETM 19A	[73]	2+1+1	5.1	3.6	Finite-volume effects not estimated.

Table 179: Finite-volume effects in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
Mainz 24	[54]	2+1	4.1, 4.8, 6.1, 4.7	4.7, 5.4, 4.0, 4.2	A number of simultaneous a , M_π and $M_\pi L$ fits are performed using term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$. The final results are obtained from a weighted average.
LHPC 24	[74]	2+1	5.6, 5.9	3.9, 4.0	Finite-volume effects are not estimated.
NME 21A	[75]	2+1	4.1, 3.0–5.8, 3.5–5.1	5.9, 5.1, 4.3	Fit performed using a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a , M_π and $M_\pi L$.
NME 20	[76]	2+1	4.1, 3.0–5.8, 3.5–5.1	5.9, 5.1, 4.3	Fit performed using a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a , M_π and $M_\pi L$.
Mainz 19	[77]	2+1	2.8–4.1, 2.4–3.6, 2.1–4.1, 2.4–3.2	4.7, 5.3, 4.2, 4.3	Extrapolation performed including a term of the form $M_\pi^2 e^{-M_\pi L} / \sqrt{M_\pi L}$ as part of a simultaneous fit in a^2 , M_π and $M_\pi L$.
χ QCD 18A	[78]	2+1	4.6, 2.7, 5.5, 2.6	4.0, 4.4, 3.9, 4.0	Extrapolation performed including a term of the form $e^{-M_\pi L}$ as part of a simultaneous fit in a^2 , M_π and $M_\pi L$.
LHPC 12A	[79]	2+1	2.8–5.6, 2.9	4.2, 4.6	Finite-volume effects investigated and found to be negligible.
LHPC 10	[80]	2+1	2.5–3.5	3.7	Finite-volume effects included in chiral fit formula and found to be negligible.
RBC/UKQCD 10D	[81]	2+1	2.7	4.6	No uncertainty estimated. Comparison between two volumes for the three heaviest pion masses shows no deviation within statistical errors.

Table 180: Finite-volume effects in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
RQCD 18	[82]	2	2.6, 1.7-4.6, 1.9-2.9	3.7, 2.8, 3.8	No significant finite-volume effects observed.
ETM 17C	[83]	2	4.5	3.1	Single ensemble
ETM 15D	[84]	2	4.5	3.0	Single ensemble
RQCD 14A	[85]	2	2.6, 1.7-4.6, 1.9-2.9	3.7, 2.8, 3.8	No significant finite-volume effects observed.

Table 181: Finite-volume effects in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	Ren.
ETM 22	[53]	2+1+1	RI'-MOM
PNDME 20A	[71]	2+1+1	RI'-MOM
ETM 20C	[72]	2+1+1	RI'-MOM
ETM 19A	[73]	2+1+1	RI'-MOM
Mainz 24	[54]	2+1	RI-MOM
LHPC 24	[74]	2+1	RI'-MOM/RI-SMOM
NME 21A	[75]	2+1	RI'-MOM
NME 20	[76]	2+1	RI'-MOM
Mainz 19	[77]	2+1	RI-MOM
χ QCD 18A	[78]	2+1	RI-MOM
LHPC 12A	[79]	2+1	RI-MOM
LHPC 10	[80]	2+1	RI-MOM
RBC/UKQCD 10D	[81]	2+1	RI-MOM
RQCD 18	[82]	2	RI'-MOM
ETM 17C	[83]	2	RI'-MOM
ETM 15D	[84]	2	RI'-MOM
RQCD 14A	[85]	2	RI'-MOM

Table 182: Renormalization in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1 + 1$, $N_f = 2 + 1$ and $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	τ [fm]	Description
ETM 22	[53]	2+1+1	[0.6–1.6] [0.6–1.5] [0.5–1.2]	Two-state fit to all τ . A comparison is made with plateau fits and the summation method.
PNDME 20A	[71]	2+1+1	[0.8–1.4] [1.0–1.7, 1.0–1.7] [0.9–1.4, 0.9–1.4, 0.9–1.4] [1.0–1.4, 0.9–1.3]	Fits to the τ - and t -dependence of three-point correlators using two or three lowest-lying states.
ETM 20C	[72]	2+1+1	[0.6–1.6]	Two-state fit to all τ . A comparison is made with plateau fits and the summation method.
ETM 19A	[73]	2+1+1	[0.6–1.6]	Two-state fit to all τ . A comparison is made with plateau fits and the summation method.

Table 183: Control of excited-state contamination in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
Mainz 24	[54]	2+1	[0.4–1.5, 0.4–1.5, 0.4–1.5, 0.4–1.5] [0.3–1.5, 0.3–1.5, 0.3–1.5] [0.3–1.4, 0.3–1.4, 0.3–1.4, 0.3–1.4, 0.3–1.4, 0.3–1.4] [0.2–1.4, 0.2–1.4, 0.2–1.4]	Two-state summation method.
LHPC 24	[74]	2+1	[0.3–1.4] [0.9–1.5]	Two-state fits to the three-point function and the summation method are utilised, with the final results obtained from a weighted average.
NME 21A	[75]	2+1	[1.3–1.8] [1.1–1.5, 1.3–1.7, 1.1–1.5, 1.1–1.5] [1.1–1.4, 1.2–1.5]	Fits to the τ - and t -dependence of three-point correlators using two or three lowest-lying states.
NME 20	[76]	2+1	[1.3–1.8] [1.1–1.5, 1.3–1.7, 1.1–1.5, 1.1–1.5] [1.1–1.4, 1.2–1.5]	Fits to the τ - and t -dependence of three-point correlators using two or three lowest-lying states.
Mainz 19	[77]	2+1	[1.0–1.4, 1.0–1.4, 1.0–1.4] [1.0–1.5, 1.0–1.5] [1.0–1.4, 1.0–1.4, 1.0–1.4, 1.0–1.4] [1.0–1.4, 1.0–1.3]	Fits to the τ - and t -dependence of correlator ratios using the two lowest-lying states.
χ QCD 18A	[78]	2+1	[0.7–1.5]	two-state fits to the three-point function.
LHPC 12A	[79]	2+1	[0.9–1.4, 0.9–1.4, 0.9–1.4, 0.9–1.4, 0.9–1.4] [0.9–1.4]	Fits to the leading (ground state) τ -dependence of summed correlator ratios.
LHPC 10	[80]	2+1	[1.1, 1.1, 1.1, 1.1, 1.1–1.2, 1.1]	Plateau fits of correlator ratio at $\tau = 1.1$ fm. Larger source-sink separation on one ensemble as cross check.
RBC/UKQCD 10D	[81]	2+1	[1.4, 1.4, 1.4, 1.4]	Single source-sink separation considered.

Table 184: Control of excited-state contamination in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
RQCD 18	[82]	2	Not given	Analysis limited by statistics
ETM 17C	[83]	2	[0.9–1.5]	Result from plateau method with $\tau/a = 14$. Consistent with estimates from summation and two-state fit methods.
ETM 15D	[84]	2	[0.9–1.3]	Result from plateau method with $\tau/a = 14$. Consistent with estimate from the summation method.
RQCD 14A	[85]	2	Not given	Plateau value at larger τ/a consistent with two-state fit.

Table 185: Control of excited-state contamination in determinations of the isovector unpolarised, helicity and transversity second moments with $N_f = 2$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

C.8 Notes to Sec. 11 on scale setting

Collab.	Ref.	N_f	a [fm]	Description
TUMQCD 22	[86]	2+1+1	0.15294, 0.12224, 0.08786, 0.05662, 0.0426, 0.03216	MILC ensembles from on-shell Symanzik-improved gauge action and rooted HISQ fermions
ETM 21	[7]	2+1+1	0.069, 0.079, 0.097	

Table 186: Continuum extrapolations/estimation of lattice artifacts in scale determinations with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Hudspith 24	[87]	2+1	0.098, 0.085, 0.075, 0.064, 0.049, 0.039	NP $\mathcal{O}(a)$ -improved Wilson fermions with tree-level Symanzik improved gauge action.
Asmussen 23	[88]	2+1	0.085, 0.075, 0.064, 0.049, 0.039	NP $\mathcal{O}(a)$ -improved Wilson fermions with tree-level Symanzik improved gauge action.
RQCD 22	[67]	2+1	0.098, 0.085, 0.075, 0.064, 0.049, 0.039	NP $\mathcal{O}(a)$ -improved Wilson fermions with tree-level Symanzik improved gauge action.
RBC/Bielefeld 07	[89]	2+1	0.3 - 0.05	Simulations with improved staggered fermions (p4fat3-action: smeared 1- link term and bent 3-link terms) at 27 different values of β .

Table 187: Continuum extrapolations/estimation of lattice artifacts in scale determinations with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	$M_\pi L$	Description
TUMQCD 22	[86]	2+1+1	129	3.25 – 4.17	At the physical point.
ETM 21	[7]	2+1+1	134.2	3.78	

Table 188: Chiral extrapolation and finite-volume effects in scale determinations with $N_f = 2 + 1 + 1$ quark flavours. We list the minimum pion mass $M_{\pi,\min}$ and $M_\pi L \equiv M_{\pi,\min}[L(M_{\pi,\min})]_{\max}$ is evaluated at the maximum value of L available at $M_\pi = M_{\pi,\min}$.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	$M_\pi L$	Description
Hudspith 24	[87]	2+1	127/131	3.51/4.05	At $m = m_{\text{symm}}$.
Asmussen 23	[88]	2+1	127/131	3.51/4.05	At $m = m_{\text{symm}}$.
RQCD 22	[67]	2+1	127/131 200	3.51/4.05 4.14	At $m = m_{\text{symm}}$. At $\tilde{m}_s = \tilde{m}_{s,\text{phys}}$
RBC/Bielefeld 07	[89]	2+1	220	5.456	

Table 189: Chiral extrapolation and finite-volume effects in scale determinations with $N_f = 2 + 1$ quark flavours. We list the minimum pion mass $M_{\pi,\min}$ and $M_\pi L \equiv M_{\pi,\min}[L(M_{\pi,\min})]_{\max}$ is evaluated at the maximum value of L available at $M_\pi = M_{\pi,\min}$.

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